DOCUMENT RESUME

ED 300 270 SE 050 116

TITLE Teaching Mathematics with Computers 9-12.
INSTITUTION New York State Education Dept., Albany.
PUB DATE 88
NOTE 66p.; Drawings may not reproduce well.
AVAILABLE FROM Publications Sales Desk, New York State Education Department, Room 164 EBA, Washington Avenue, Albany, NY 12234 ($1.50).
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS *Computer Oriented Programs; *Computer Software; *Course Descriptions; Curriculum Development; *Mathematics Curriculum; *Mathematics Instruction; Microcomputers; Problem Solving; Secondary Education; *Secondary School Mathematics; State Departments of Education; Teaching Methods
IDENTIFIERS *New York

ABSTRACT

This publication is designed to show how computers can be used effectively in secondary school mathematics curricula. Section I provides a description of the various types of software that could be incorporated into the secondary school mathematics curriculum. Procedures for evaluating the software and using it in a classroom or laboratory situation are discussed. Section II presents a model for the infusion of an ongoing component of computer instruction into New York State curricula. The focus is on a metacognitive approach to problem solving; examples are presented in the multi-step computer solutions to problems. Section III addresses the New York State Regents Action Plan which calls for a required second year of mathematics by presenting a computer mathematics course as an option for a local diploma. A list of topics is presented as a guide for course development, with a detailed lesson plan for one topic. Section IV proposes an advanced computer mathematics course, with suggested topics and approach. Section V provides two tables referencing software to specific content topics. Included in the appendices are two computer programs which teachers might use to illustrate the relevancy of using computers to solve real-world problems. (MNS)

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TEACHING MATHEMATICS WITH COMPUTERS

9-12

10 PRINT "ROLLING A 2"
20 PRINT "WITH A SINGLE DIE"
30 C=0
40 FOR T=1 TO 50
50 D=INT(6*RND(1))+1
60 IF D=2 THEN C=C+1
70 NEXT T
80 PRINT"A 2 WAS ROLLED"; C; "TIMES."
90 END

The University of the State of New York
The State Education Department
Bureau of Mathematics Education
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Albany, New York

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ASSISTANT COMMISSIONER FOR ESC EDUCATION PLANNING AND SUPPORT SERVICES
DIVISION FOR PROGRAM DEVELOPMENT

TO: Persons With Responsibility for Implementing Mathematics Programs, 9-12

FROM: Edward T. Lalor, Director
Division for Program Development

Charles J. Trupia, Director
Division for General Education

Teaching Mathematics With Computers, 9-12 is designed to show how computers can be used effectively in secondary school mathematics curricula. This publication describes various types of software suitable for use at the secondary level and tells how they can be integrated into the curriculum. The publication also describes the infusion of computer instruction into current State syllabi and provides guidelines for the development of introductory and advanced computer math courses.

Teaching Mathematics With Computers, 9-12 has been sent to principals of public and nonpublic secondary schools; director and coordinators of curriculum and instruction; secondary school mathematics teachers - grades 8-12, directors, coordinators, supervisors, and mathematics department chairpersons; and directors and coordinators of instructional technology.

Additional copies of this publication can be obtained by submitting requests in writing on school stationary to the:

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New York State Education Department
Room 164 EBA
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Albany, NY 12234

TEACHING MATHEMATICS WITH COMPUTERS 9-12

The University of the State of New York
The State Education Department
Bureau of Mathematics Education
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1988

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FOREWORD

Teaching Mathematics With Computers, 9-12 has been prepared for school administrators, curriculum specialists, mathematics supervisors and teachers, and others responsible for implementing curriculum. The publication provides suggestions for using computers to enhance student understanding of mathematical concepts and applications in the secondary school mathematics curricula.

This publication was prepared under the direction of Benjamin Lindeman, Associate, Bureau of Mathematics Education and Carol Anne Stiglmeier, Associate, Bureau of Curriculum Development.

Coordinating writer for this publication was Gerald Elgarten, Assistant Professor, City College, New York, New York. Cooperating writers were: Valarie Elswick, Mathematics Teacher, Roy C. Ketcham High School, Wappingers Falls School District; Jeffrey Hering, Supervisor, Cortland City Schools, Cortland, New York; and Rita Petosa, Teacher of Mathematics and Computer Science, Adrien Block Intermediate School 25, Community School District 25, Queens, New York.

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INTRODUCTION

The impetus for this project came from the New York State Regents Action Plan and local needs assessments as well as recommendations from professional societies and associations. The Conference Board of the Mathematical Sciences, in a September 1982 meeting, addressed the topic "THE MATHEMATICAL SCIENCES CURRICULUM K-12: WHAT IS STILL FUNDAMENTAL AND WHAT IS NOT." Among their many suggestions and recommendations are:

- Students should use calculators and computers both to develop concepts and to do tedious computation.
- Students should avoid the tedious repetition of systematic methods for solving problems by doing them a few times and then programming a computer to do them.
- Students who are not necessarily talented in mathematics and not headed for careers in science or technology will gain a fresh outlook on mathematics through the use of computers in mathematical tools and instructional media.
- Students should use computer graphics packages to get a visual sense of geometric concepts and transformations.

These suggestions and recommendations for increased use of computers in the mathematics curricula are supported by the National Science Board Commission on Precollege Education in Mathematics, Science and Technology in the report, "EDUCATING AMERICANS FOR THE 21ST CENTURY," and in the National Council of Teachers of Mathematics report, "AN AGENDA FOR ACTION: Recommendations for School Mathematics of the 1980s."

This five-part publication is based upon the recommendations of the aforementioned reports. It is designed to make suggestions and give examples of how computers may be used to enhance student understanding of mathematical concepts and applications in the secondary school mathematics curricula.

Section I provides a description of the various types of software that could be incorporated into the secondary school mathematics curriculum. Procedures for evaluating the software and using it in a classroom or laboratory situation are discussed.

Section II presents a model for the infusion of an ongoing component of computer instruction into *General High School Mathematics* and the *Three-Year Sequence For High School Mathematics*. The focus is on a metacognitive approach to problem solving. That is, students will be taught what they should be thinking about when solving problems. Examples of this approach are presented in the multi-step computer solutions to problems.

Section III addresses the New York State Regents Action Plan call for a required second year of mathematics by presenting a computer mathematics course as an option for a local diploma. Such a course should include mathematics lessons in a regular classroom setting as well as related computer activities in a laboratory. Several school districts have developed computer course outlines that the State Education Department has approved for second-year mathematics credit. A list of topics from these approved courses is presented as a guide for the development of your own computer mathematics course. A detailed lesson plan for one of the topics is presented. This will highlight how a metacognitive approach to problem solving may be used. Scheduling students for classroom and computer laboratory lessons, as well as suggestions for grading, is also discussed.

Section IV proposes an advanced computer mathematics course that can be used as one of the courses in a five-year sequence in mathematics. The suggested topics and approach are an outgrowth of an extensive review of the literature as it relates to the impact of technology on mathematics and computer education.

Section V provides two tables that can be used to facilitate the selection and use of software within the curriculum. Table 1 is a numbered alphabetical listing of software. Subject and specific content topics from the high school curriculum are outlined in Table 2. The software in this table is referenced to the topics by the number used in the alphabetical listing of Table 1. This part also contains specific suggestions for using commercially prepared software in mathematics instruction (General Mathematics and Sequential Mathematics, Course I, Course II, and Course III).

Included in the Appendices are two computer programs which teachers might use to illustrate the relevancy of using computers to solve real world problems. Both programs relate to the environmental concerns and might be used in relating mathematics to science and/or ecology. The programs were written by Richard A. Wolfert, a teacher at PS 309, Brooklyn.

This document is meant to be a guide, not a prescription. As more experience is developed in using computers in the classroom, new information will form the basis for an update of this document.

SECTION I

Using Software in Secondary School Mathematics

SOFTWARE

Software, sometimes referred to as courseware, is that material which instructs the computer to perform certain tasks. In this document, software refers to programs that are suitable for use in reviewing and developing mathematical skills and concepts. Software programs may also provide for additional practice, applications of various concepts, and testing. In general, the term *software* is used when referring to any computer program.

The following categories characterize the instructional uses of software in the mathematics classroom:

Drill and Practice	Computer Managed Instruction
Simulation	Computer Assisted Instruction
Demonstration	Programming
Problem Solving	Applications
Tutorial	

Types of Software

1. Drill and Practice

   Drill and practice usually involves the presentation of a question, student response, feedback, and recycling of the process with a new question based on the student's response.

2. Simulation

   The computer can be used to simulate real events and situations which could not otherwise be viewed in a classroom setting. Games; economic and business problems; environmental models; experiments in biology, physics, and chemistry; and concepts in mathematics can be illustrated with time, size, and other variables controlled through computer simulation.

3. Demonstration

   Mathematical concepts and graphics representations can be demonstrated in a meaningful manner with the computer. Limits, families of curves, area approximations, probability, and number concepts are but a few of the topics that can be illustrated with either commercially prepared programs or teacher-student developed programs. Sound and color graphics can be employed to focus student attention on key points.

4. Problem Solving

   Using the computer as a tool for problem solving in mathematics includes employing the computer as a high-speed calculator, using graphing programs to generalize concepts and to evaluate functions, and using programs to develop inductive reasoning skills. Writing a computer program in itself represents a most meaningful model in the problem-solving process.

5. Tutorial

   Tutorials usually consist of the presentation of knowledge in either a direct or interactive step-by-step format which is then followed by drill and practice.

6. Computer Managed Instruction (CMI)

   Management of instruction includes diagnosing and testing, student record keeping, and instructional materials generation. Mastery learning techniques can be employed by using a diagnostic-prescriptive software package that uses tailor-made objectives to deal with each student as a unique individual.

7. Computer Assisted Instruction (CAI)

   Computer assisted instruction (CAI) is the term most often used to describe the delivery of instruction via a computer. CAI includes drill and practice, tutorial, and simulation programs.

8. Programming

   The process of programming involves defining and analyzing a problem, structuring an algorithm, and writing and revising (coding and debugging) a program. Thus, programming can be used to stimulate thinking and to reinforce student understanding of the mathematical concepts under consideration.

9. Applications

   Three types of computer software applications which can be used to enhance mathematical concepts and algorithms are spreadsheet, data base management system, and word processing programs.

A *spreadsheet* program allows students to work with numeric information arranged in columns and rows. Formulas or rules may be applied to specific rows and/or columns. Once a piece of data is changed on the spreadsheet, all associated values on the spreadsheet matrix are changed (recalculated) accordingly.

A *data base management* system is a program that would enable students to work with large quantities of data that can be stored within the system (e.g., collecting endangered species data or information on pollution levels).

A *word processing* program, although not a specific mathematical tool, allows students the opportunity to easily communicate their thoughts, strategies, and problem-solving techniques.

These three types of software are just beginning to be explored and used as tools in the mathematical environment. The student does not need to know a specific programming language to use these programs. Skills learned with these tools can also be applied to other subject areas as well.

Evaluation and Acquisition Criteria

It is very important to evaluate computer software before making a decision to purchase it. If possible, the software should be previewed and used by those for whom it is intended. A record of the previewer feedback should be kept. This record, along with a written evaluation, should be considered when reaching the final decision of whether or not to purchase the software.

1. Previewing Process – How and by Whom?

   Some commercial companies will send software to a school district on a preview basis for a fixed number of days. Try to obtain the software you wish to preview from the commercial company or distributor first. If it is not available on a preview basis, then other possibilities need to be explored. Software may be borrowed from a local library system that has computer materials, from a local BOCES, or from a nearby school district. Another possibility is to make arrangements with a local distributor or computer store to try out the software at their facility.

   As many people as possible and reasonable should be involved in the previewing process. Previewers from a school district might include the following: students, teachers, a department chairperson, a curriculum specialist, or a media specialist. If the previewing process is conducted at a school, more people may be involved than when the process is conducted at a local computer store.

2. Evaluation – A Result of Previewing

   A written record of evaluation of any previewed software should be maintained. Reference to these records will prevent additional time being spent to evaluate a piece of software that was previously rejected. If a software package has recently been updated, then the evaluation report on file can be used when previewing it again to help determine whether or not the previous areas of weakness still exist.

   A format needs to be established for collecting previewer feedback. It can be as simple or complex as your particular needs dictate. An evaluation form may contain a general checklist of items or criteria that you want the software to meet. Are the displayed instructions clear, concise, and accurate? Is the format of the displayed material easy to read? Is the material presented unbiased and free of stereotypes? Are critical thinking skills being developed? These are just a few of the many questions you might want answered in your evaluation process.

   The written evaluation may call for an explanation of particular strengths or weaknesses. It may ask a teacher to note observations of student use. Do the students choose to use the software? Is it easy for them to use and understand, yet still challenging to them? Are the students able to accomplish the objectives set by the teacher with this software? The evaluation form could also ask the previewer to note if purchase of the software is recommended and the reasons why.

   An evaluation form developed for this publication is presented on the next two pages.

Many different evaluation forms exist. You may find one that fits your needs perfectly or you may want to adapt portions from those available to make your own form. Several sources of other evaluation forms follow.

- CONDUIT Package Evaluation Form For Microcomputer-Based Instructional Materials, CONDUIT, P.O. Box 388, Iowa City, Iowa 52244
- *Developing Minds: A Resource Book for Teaching Thinking*, pgs. 258-264, Association for Supervision and Curriculum Development, 225 N. Washington Street, Alexandria, Virginia 22314
- Microcomputer Educational Materials: Evaluation, Minnesota Educational Computing Consortium, 2520 Broadway Drive, St. Paul, Minnesota 55113
- Microcomputer Software Evaluation Form, EPIE Institute, P.O. Box 620, Stony Brook, New York 11790
- Software Evaluation, Selection and Management Guidelines for School Districts, CNY Instructional Computer Network, Onondaga/Madison BOCES Re-

SAMPLE SOFTWARE EVALUATION FORM

Program Title: _____

Reviewer: _____ Date: _____

Package Title: _____ Copyright Date: _____

Publisher: _____ Price:

Quantity discount available: _____YES _____NO

System Requirements:

Apple: ___II ___II+ ___IIe ___IIc ___Macintosh

IBM: ___PC Jr ___PC ___XT ___AT

Tandy: _____

Commodore: _____

Other: _____ (type)

Memory: ___48k ___64k ___128k ___256k ___512k
Other: _____ (amount)

Input: ___Diskette(__SS or __DS) __Cassette __Cartridge __Joystick/Paddles
__# of Disk Drives needed Other:_____

Output: __Text-only Printer __Graphics-capable Printer
__Monochrome Monitor __Color Monitor

Can software be NETWORKED? __NO __YES Cost: ____ System type:_____

Program/Package Type (check all that apply):
___Tutorial ___Drill & Practice ___Simulation ___Instructional Game
___Problem Solving ___Demonstration ___Management ___Informational
Other:_____

Subject for which software is appropriate (check all that apply):
___Math 7 ___Math 8 ___Gen. Math ___Course I ___Course II ___Course III
___Calc ___Pre-Calc ___12th Yr. Math Other:_____

Appropriate for use by:
___Individual ___Small Group (2-8) ___Large Group/Class ___Classroom Network

Can be taught by:
___Teacher ___Aide ___Parent ___Student

Support Materials Available:
___Documentation for Teacher ___Documentation for Student ___Worksheets
___Workbook ___Textbook:_____ ___Other diskettes:_____
Is written documentation needed to use the program diskettes? ___NO ___YES

In General:
Software is easy to use: ___YES ___NO
# of Lessons/Programs on a Diskette:_____
Approximate working time: _____per Lesson/Program _____per Diskette

SAMPLE SOFTWARE CONTENT EVALUATION

Program Title:_____ Reviewer:_____

GIVE YOUR RESPONSE BASED UPON USE OF PROGRAM DISKETTES ONLY.		YES	NO	N/A
A.	Instructional objectives are clearly defined.	___	___	___
B.	Content is free of mechanical errors.	___	___	___
C.	Content is free of grammatical errors.	___	___	___
D.	Content is socially balanced, free of bias, and free of stereotypes.	___	___	___
E.	Frame formatting is well done.	___	___	___
F.	Frame format is consistent and easy to read.	___	___	___
G.	Directions/instructions are provided.	___	___	___
H.	It is possible to review the directions/instructions.	___	___	___
I.	Feedback is immediate.	___	___	___
J.	Feedback remediates.	___	___	___
K.	Positive feedback is randomly generated.	___	___	___
L.	Random generation is used in the lessons/activities.	___	___	___
M.	Help or hint options are available.	___	___	___
N.	Use of graphics and/or sound is effective.	___	___	___
O.	Student is able to control pace and sequence of program.	___	___	___
P.	It is possible to exit the program/lesson at any time.	___	___	___
Q.	Menu allows choice of lesson.	___	___	___
R.	Menu allows choice of difficulty level.	___	___	___
S.	Student receives evaluation/score at end of program/lesson.	___	___	___
T.	Program is interactive.	___	___	___
U.	Teacher can change the program content.	___	___	___
V.	Program requires little preparation time.	___	___	___
W.	Program maintains high student motivation.	___	___	___
X.	Content is appropriate for remediation.	___	___	___
Y.	Content is appropriate for enrichment.	___	___	___
Z.	Software is "bomb proof" and "error trapped."	___	___	___

Program Strengths:_____

_____

Program Weaknesses:_____

_____

Overall Rating of Software: ___Excellent ___Very Good ___Good ___Fair ___Poor

Additional Comments:_____

_____

gional Computing Center, 6820 Thompson Road, Box 4866, Syracuse, New York 13221

Other sources of written evaluations that should not be overlooked are the software reviews contained in most professional and commercial publications. Such articles can be added to the other written feedback you have and act as support for the acquisition decision. Several sources are listed here.

Professional Sources

- *Arithmetic Teacher*, 1906 Association Drive, Reston, Virginia 22091
- *Mathematics Teacher*, 1906 Association Drive, Reston, Virginia 22091
- *School Science and Mathematics Journal*, School Science and Mathematics Association, Inc., 126 Life Science Building, Bowling Green State University, Bowling Green, Ohio 43403
- *The Computing Teacher*, International Council for Computers in Education, 1787 Agate, University of Oregon, Eugene, Oregon 97403

Commercial Sources

- *Classroom Computer Learning*, 2451 E. River Road, Dayton, Ohio 45439
- *Electronic Learning*, Scholastic, Inc., 730 Broadway, New York, New York 10003-9538
- *InfoWorld*, CW Communications, Inc., Popular Computing, Inc., 1060 Marsh Road, Suite C-200, Menlo Park, California 94025-9538
- *MacUser*, MacUser Publications, Inc., 25 West 39th Street, New York, New York 10018
- *Teaching and Computers*, Scholastic, Inc., 730 Broadway, New York, New York 10003

3. Acquisition – Yes or No?

   An individual or a committee can make the decision to purchase software, depending on the circumstances involved. All aspects of the evaluative process should be considered during the decision process. Acquisition of software should not be based solely on a favorably written evaluation, but also on observations of its use in the classroom and laboratory. The software that is purchased should certainly follow sound educational principles and should meet the objectives of the course for which it is being selected.

Environment for Use of Software

Where in your building or school district will software be best used? Should it be used only for demonstration purposes? Can it be used with a small group, a large group, or a full class? Is it possible to network the software? Can it be used by individuals in a computer lab, a library, or a media center? Is the necessary hardware available for using the software? What is the cost of purchasing the additional hardware that might be needed? These are some of the questions that should be addressed as part of your software evaluation process. The answers will vary depending on the particular software package being evaluated. Suggestions for management and supervision of software use are presented next.

Management and Supervision of Software and Hardware

Once software is available for students and teachers to use, appropriate management and supervision are needed. The most typical locations for both computer hardware and software are the classroom, the computer lab, the school library, the media center, and the mathematics department office. Many variations are possible depending on the physical layout of the school building, the origin of the funding, and the district policy of who is in charge – a curriculum specialist, a computer coordinator, a media coordinator, a department chairperson, or the mathematics teacher. Possibilities for typical locations and management of both software and hardware are discussed in this section.

Software

Some of the questions that should be considered for software management follow. Where is it kept? Who is responsible for the software? How does a teacher know what is available? Is there a sign-up procedure for using the software? Are multiple copies available?

1. Locations

   To avoid theft, security should be maintained when storing software. To prevent unnecessary damage, software should not be exposed to extreme heat and cold nor magnetic fields. The location should also be free of dirt and dust.

   a. Classroom

      Teachers who are assigned software could store it in their classrooms. They could use an open shelf, a lockable cabinet, or a file drawer to house the software.

   b. Computer Lab

Some schools may elect to keep their software in the computer lab. It might be stored in cabinets with locks or on open shelves.

c. Library/Media Center

Currently, many libraries and media centers have computer stations and software available for use. The librarian or media center coordinator is responsible for the use of the computer materials.

d. Mathematics Department Office

If the mathematics department office has software available, then the department chairperson could supervise its storage and use.

2. Management

Depending on the size and needs of a school, one or several people may be in charge of software. For instance, all software might be kept in the library or media center under the librarian's supervision. Perhaps the mathematics department chairperson might be responsible only for the software used by the department. Or, a teacher aide in a computer lab might have responsibility for the software stored there.

There are many possibilities and many levels of responsibility. Ultimately, the teacher who is going to use the software has the final responsibility for supervising its use.

All teachers should be aware of the software materials available within their school district. This is best accomplished by the periodic publication of a software bibliography. An annotated bibliography could include information about the hardware needed, a brief description of the diskette contents, indication of software type (drill and practice, simulation, etc.) and grade level. The content of the bibliography should be designed to promote efficient use by both students and teachers.

a. Classroom

The teacher usually assumes responsibility for managing and supervising the use of software in the classroom. Prior to a lesson, the teacher needs to make sure that the software is operational and that its use is appropriate for the classroom setting.

b. Computer Lab

A computer laboratory can serve several purposes for a mathematics teacher. A teacher can take an entire class to the computer lab for instruction and demonstrations involving all types of software, or the teacher can give homework assignments to be completed in the lab.

The teacher usually makes arrangements for the software to be available in the lab regardless of whether the lab is supervised by the teacher or by a teacher aide.

c. Library/Media Center

The use of software in the library or media center is usually under the direction of the librarian. A sign-out procedure is needed, just as with other media materials, even when the software is going to be used in that location. A few libraries allow students to take software home over the weekend, but this is not recommended when it might conflict with copyright and leasing agreements.

d. Mathematics Department Office

A sign-up and sign-out procedure is a good idea when the mathematics department software is stored in its office. Teachers need to plan ahead for the availability of the software to avoid conflict with the plans of others. If a particular piece of software is in high demand, it might be a good idea to have several copies available.

Hardware

The term *hardware* is used to refer to the computer equipment necessary for the use of computer software. The usual equipment consists of the CPU, the keyboard, the disk drives, and a monitor. Other equipment, which can be used with the usual computer set-up, is referred to as peripheral equipment. A printer, a plotter, a modem, a mouse, joysticks, paddles, and a voice synthesizer are just a few examples of peripherals.

Hardware considerations need to accompany both the software evaluation/acquisition process and the final methods of teacher use. Where is the hardware kept? What types of security are needed? Who is responsible for its use and repair? How much hardware is available? Is there a portable computer station that could be moved from room to room? Are large-screen monitors available for classroom demonstrations? Are printers and other peripherals available? These are just a few of the questions that should be considered. A description of possible locations for computer hardware and its management follows.

1. Location

The size of a school and its sources of funding are key factors in determining how much hardware will be available and where it will be kept. Suggestions for possible locations and set-ups are given here.

a. Classroom

Several options are possible for hardware in the

classroom. A mobile computer station could be brought into the classroom when needed. A mathematics classroom could contain one or more computer stations. A mobile computer station or a permanent station in the classroom should also include a printer. If the classroom contains more than one computer station, then ideally there should be one printer for each station.

b. Computer Lab

The size of the rooms used for computer labs will vary from one school to another and, therefore, have an effect on the amount of computer equipment they can hold. Each computer station in the lab will vary, depending upon the lab's configuration – stand-alone microcomputer stations, a networked system of microcomputers, or terminals. A stand-alone microcomputer station, in addition to the usual equipment, should have a printer as well as a variety of other peripheral devices. A networked system should be supported by a high-speed printer.

c. Library/Media Center

One or more computer stations may be found in most school libraries or media centers. It is a good idea to have at least one printer available in this location. Other peripherals might also be available from this source if the library media specialist has responsibility for all the computer equipment.

d. Mathematics Department Office

The mathematics department office is usually a good location for at least one permanent computer station, including printer, available for teacher use throughout the school day. The office could also maintain other peripheral equipment used specifically by mathematics teachers.

A mobile computer station might also be made available from the mathematics office, if classrooms do not have permanent computer stations. A mobile set-up should include a printer and a large-screen monitor, especially for demonstration purposes.

2. Management

For maximum and efficient use of computer hardware, many concerns need to be addressed. Who is in charge of seeing that repairs are made? Is it the principal, a computer coordinator, the mathematics department chairperson, or the librarian? Who makes the repairs? Who receives reports of malfunctions and needed repairs? These concerns need to be dealt with quickly to avoid long delays of equipment availability to teachers and students.

What types of security are needed? Should the equipment be bolted down to permanent desks and table tops? Do rooms have bars across the windows and special non-master locks on the doors? Who will have keys to these locks? Policies and procedures should be established for the school building and the various locations within it where equipment is kept.

Who is responsible for scheduling the use of computer hardware? Has consideration been given to some kind of sign-up procedure for both students and teachers? Location of the equipment is sometimes the determining factor as to who will be responsible for it. Some possibilities for management and supervision follow.

a. Classroom

In the classroom, the teacher has responsibility for the hardware that is available and needed for instruction. The manner in which software can be used in the classroom is dependent upon the amount of hardware present. If mobile equipment is used, then advanced scheduling and sign-up procedures might be required. Careful and creative planning is also necessary.

b. Computer Lab

Activities in a computer lab may be supervised by a teacher aide or by a teacher. Rules governing the use of the facilities need to be established and should be posted to avoid conflicts. Many variations are possible. For example, students could sign up one week in advance, for either a full or half period for their individual work, not to exceed a set weekly limit. Teachers wishing to take their classes to the computer lab could be asked to sign up either one or two weeks in advance.

c. Library/Media Center

In this environment the librarian is usually responsible for supervision of hardware use. If equipment is limited, a sign-up procedure similar to the one used for a computer lab may be helpful.

d. Mathematics Department Office

If mobile computer equipment used in the mathematics classrooms comes from this office, then the chairperson has the responsibility of establishing a sign-up procedure for its use and return.

e. Other

The use of mathematics can easily cross the boundary into diverse subject areas, such as environmental studies (e.g., the collection and analysis or forecasting of a species population growth data base). In such cases, mobile or permanent stations will be supervised by the appropriate teacher or supervisor.

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SECTION II

Infusion of Computer Instruction Into Current New York State Mathematics Syllabi

• GENERAL HIGH SCHOOL MATHEMATICS

Introduction

This section is designed to provide a model for the infusion of computer instruction into the New York State Syllabus, *General High School Mathematics*. The model formulates how the computer can be used as a tool to support the contents of the syllabus. Fundamental considerations to be addressed are given below.

1. It is assumed that students come to this course with little or no experience in programming.
2. For students who, at this point, find the structure necessary for learning programming cumbersome, a goal in this course is to help them understand how a computer follows instructions. A more formalized approach might be taken in a more advanced course. While the nature of the computer program should be considered, the major focus in using computers should be directed toward reviewing, reinforcing, and extending topics covered in the *General High School Mathematics* syllabus.
3. The material outlined in this section is suggested as a supplement to the use of commercially produced software, not as a substitute for it.
4. A computer laboratory setting would be the most effective environment for this component of *General Mathematics*. A ratio of at most two students per computer is suggested.
5. Computer mathematics textbooks would provide an excellent resource for the students. Otherwise, the use of teacher-prepared materials is suggested.

Some sample lesson segments are listed below and are intended to provide guidance for the classroom teacher.

Lesson A

This lesson serves as a model to illustrate how lessons can be provided throughout the school year to help remediate the students' weaknesses in basic arithmetic skills.

i) Some discussion of the variables used in BASIC as well as arithmetic symbols should precede these lessons. Students may be asked to translate equations into BASIC as follows.

Mathematics	BASIC
$P = 4S$	P = 4*S
$A = X^3$	A = X ^ 3
$Y = A \div B + 2AB$	Y = A/B + 2*A*B
$M = NP - X \div Y$	M = N*P - X/Y

ii) Students should then be taught to trace a program. Tracing a program is designed to help students understand how the computer follows instructions. Each line of the program is analyzed as the student is asked to record its effect on the screen and/or in memory as follows.

Screen Display	Memory
10 INPUT S	S | P
20 P = 3*S	4 |
30 PRINT P	| 3*4 = 12
40 END	

RUN ⟶ The word RUN allows the computer to follow the instructions.

?4 ⟶ The word INPUT stops the computer and has it wait for the user to put a value into memory cells by typing it next to the question mark and pressing RETURN (ENTER).

12 ⟶ Stress that in line 20 the computer only calculates the value of 3 times the value stored in memory cell S and stores it in memory location P. It is only when the computer executes line 30 that the calculated value (12) is displayed.

iii) In order to generate the trace, the following questions can be asked.

- What word do we have to type in order to get

the computer to follow (execute) the instructions (program)?

RUN

- What appears on the screen when the computer executes line 10 which has the word INPUT?

  A question mark

- What is the computer waiting for you to type after the question mark?

  A value for S

- What does the computer display as a result of executing (doing) line 20?

  Nothing

- Where would you find evidence that line 20 has been executed?

  In memory cell P

- What does the instruction PRINT P cause the computer to do?

  Display the number that is stored inside memory location P.

- What is the name of the memory location where the value of 3*4 is stored?

  P

- What does line 40 indicate to the computer?

  There are no additional instructions.

iv) Ask the students to trace the program with values which will reinforce arithmetic skills; e.g.,

Screen Display	Memory	
RUN	S	P
?6.8	~~6.8~~	
20.4		3*6.8 = ~~20.4~~
RUN		
?2500[a]	~~2500~~	
7500		3*2500 = ~~7500~~
RUN		
?.5[b]	.5	
1.5		3*.5 = 1.5

Note that the cross-out shows that 6.8 is no longer stored in memory cell S but has been replaced by 2500, similarly for 2500 in S and 20.4 and 7500 in memory cell P.

a. Stress that no comma should be used. When verifying their outputs on the computer, students might be asked to try the comma. Based on the output, ask them what value they think the computer used for S.

b. Ask the students to try to enter 1/2 in fractional form. Discuss with them the computer's interpretation of 1/2 as "1 divided by 2." Ask them to change the fraction to decimal form. Ask them to indicate the limitations they would have with fractions like 1/3.

v) Ask the students to verify their traces by typing and running the programs on the computer. The teacher should check that a trace has been completed before sending students to the computer.

vi) Other sample programs are listed below and serve only as illustrations. The individual needs of the students should determine the extent to which these or other programs are used. In addition to providing the students with reinforcement of arithmetic skills, the traces enable students to gain greater facility with the use of variables. In each of the first three programs illustrated, the students are asked to recognize that the values entered into the computer as a result of the execution of the INPUT statement are stored in special memory locations. These values are then displayed as a result of the appropriate PRINT statements. Notice that in the last program the students are asked to perceive that the value entered and stored is in memory cell M and is displayed when line 40 is executed, but is then replaced in memory with a calculated value (line 50). It is this new value that is displayed when line 60 is executed.

1. *Program*

```
10 INPUT A,B,C
20 PRINT A
30 PRINT B
40 PRINT C
50 D=6*A
60 PRINT D
70 E=B/C
80 PRINT E
90 END
```

a) What is the output if A=2.3, B=2.4, and C=4?

b) What is the output when A=−10, B=−3.6, and C=−0.6?

2. *Program*

```
10 INPUT E,T
20 PRINT E
30 S=E/T
40 PRINT T
50 R=2*S
60 PRINT S
70 PRINT R
80 END
```

a) What is the output if E=12.8 and T=4?

b) What is the output when E=12 and T=−3?

3. *Program*

```
10 INPUT R,P
20 PRINT R
30 E=R−P
40 PRINT P
50 PRINT E
60 END
```

a) What is the output when R=5 and P=2 2/5?

b) What is the output when R=7 and P=−2?

4. *Program*

```
10 INPUT X
20 L=2*X
30 G=3*L
40 PRINT L
50 PRINT G
60 END
```

a) What is the output when X=3.4?

b) What is the output when X=−4?

5. *Program*

```
10 INPUT K,L,M
20 PRINT K
30 PRINT L
40 PRINT M
50 M=L+K
60 PRINT M
70 END
```

a) What is the output if K=1.2, L=4, and M=5?

b) What is the output if K=−8, L=−2, and M=−5?

Lesson B

This lesson serves as a model to illustrate how students can revise programs already written in order to provide different outputs. Revision of programs provides students with another vehicle to review and reinforce their understanding of the order in which arithmetic operations are performed and algebraic expressions are evaluated.

i) The discussion of the order of arithmetic operations should precede this lesson. The students should also be familiar with the procedure outlined in Lesson A for tracing a program.

ii) Ask the students to trace and then run the program listed below when L=5 and W=7.

SCREEN DISPLAY	MEMORY		
	L	W	P
10 INPUT L,W	5	7	
20 P=2*L+W			2*5+7=17
30 PRINT P			17
40 END			
RUN			
?5, 7			
17			

iii) Ask the students if the program can be used to find the perimeter of a rectangle. Ask them to adjust the program so that it will find the perimeter of a rectangle. Ask them to change line 20 so that it reads

P=2*L+2*W or P=2*(L+W).

Trace the program with the revision in line 20 and then run the program in order to verify the predicted output; e.g.,

Screen	Memory		
RUN	L	W	P
?5, 7	5	7	2*5+2*7=24 or 2*(5+7)=24
24			24 24

iv) Other programs and desired outputs are listed below. For each, the students are asked to trace the given program, run it to verify the trace, compare the screen output to the desired output, revise the calculation in line 20, and then run the revised program to verify that the desired output is indeed produced. Again, the students' individual needs should be your guide in the selection of the programs.

1. *Program* / *Desired Output*

```
10 INPUT A,B          RUN
20 C=(A+B)*3          ?1,3
30 PRINT A            1
40 PRINT B            3
50 PRINT C            10
60 END
RUN
?1,3
```

Note: In this program, removing the parentheses in line 20 would produce the desired result.

2. *Program* / *Desired Output*

```
10 INPUT A,B,C        RUN
20 D=A+B*C            ?2,3,4
30 PRINT D            2
40 PRINT C            3
50 PRINT B            4
60 PRINT A            20
70 END
RUN
?2,3,4
```

3. *Program* / *Desired Output*

```
10 INPUT X,Y          RUN
20 A=X+Y*Z-Z          ?1,2,3
30 PRINT X            1
40 PRINT Y            2
50 PRINT Z            3
60 PRINT A            6
70 END
RUN
?1,2,3
```

4. *Program* / *Desired Output*

```
10 INPUT A,B          RUN
20 D=A-B*3            ?10,2
30 PRINT A            10
40 PRINT B            2
50 PRINT D            24
60 END
RUN
?10,2
```

v) A variation is illustrated in the problems that follow. Ask the students to insert a formula in

line 20 in order to complete a program that will produce the desired output. Then ask the students to run the programs in order to verify the outputs. Again, the students' strengths and weaknesses should guide your selection of programs and data in the desired outputs.

1. *Program*
```
10 INPUT F
20
30 PRINT S
40 END
```
Desired Output
```
RUN
?10
12
RUN
?-5
-3
RUN
?3.5
5.5
```

2. *Program*
```
10 INPUT X
20
30 PRINT Y
40 END
```
Desired Output
```
RUN
?7
15
RUN
?-2
-3
RUN
?1.5
4
```

Lesson C

This lesson serves as a model to illustrate how computers can be used to explore mathematical ideas.

i) The students should be familiar with the algorithms for finding the area of a rectangle, the perimeter of a rectangle, and the factors of a number.

ii) The students should be asked to name all pairs of integers that can be used as the measures of the dimensions of a rectangle whose area is 36 sq. cm. Ask the students to list all the factors of 36 in a systematic way as follows.

Lengths (in cm)	Widths (in cm)
36	1
18	2
12	3
9	4
6	6

iii) Ask the students to draw on graph paper the rectangles whose dimensions are listed above. Ask the students to verify that the interior of each rectangle contains 36 sq. units. Ask the students to identify the rectangle which seems to have the smallest perimeter.

iv) In order to verify quantitatively that the 6 by 6 rectangle has the smallest perimeter, present the following program.

Program
```
10  PRINT "COMPARING THE PERIMETERS OF RECTANGLES"
20  PRINT "LENGTHS", "WIDTHS", "PERIMETERS"
30  PRINT "(IN CM)", "(IN CM)", "(IN CM)"
40  READ L,W
50  IF L=0 THEN 110
60  P=2*L+2*W
70  PRINT L,W,P
80  GOTO 40
90  DATA 36,1,18,2,12,3,9,4,6,6
100 DATA 0,0
110 END
```

v) Before asking the students to run the program, trace it with them as follows. Mention that a statement other than the INPUT statement is used to enter data in this program; it is called a READ/DATA statement. This statement tells the computer to read a pair of numbers from the DATA statement in line 90 and store them in memory cells named in the READ (line 40) statement (L, W). Each time line 40 is executed, the next pair of numbers is read in line 90 until there are no more numbers and then the pair in line 100 is read. Ask the students what they think is the purpose of the DATA statement in line 100 in conjunction with line 50. Say that in order to avoid an OUT OF DATA message, two more numbers are added to our data list and the computer checks for one of them each time a pair of numbers is read. The number that is checked by the computer is called a "flag" because it signals (or flags) the computer that there are no more data. The programmer can use any number but usually uses a number that will not be mistaken for the data. Here, 0 is used since the lengths of the dimensions of the rectangles cannot be zero.

Screen
```
RUN
COMPARING THE PERIMETERS
OF RECTANGLES
```

LENGTHS (IN CM)	WIDTHS (IN CM)	PERIMETERS (IN CM)
36	1	74
18	2	40
12	3	30
9	4	26
6	6	24

Memory

L	W	P
36	1	2*36+2*1 = 72+2 =74
18	2	2*18+2*2 = 36+4 =40
12	3	2*12+2*3 = 24+6 =30
9	4	2*9+2*4 = 18+8 =26
6	6	2*6+2*6 = 12+12 =24
0	0	

Note: The values in memory cells L, W, and P change each time lines 40 and 60 are re-executed.

Ask the students to note also that the 6 by 6 rectangle, which is a square, has the smallest perimeter.

vi) Divide the students into groups of four. Ask each group of students to determine, for one of the areas, all pairs of integers that can be used as the measures of the dimensions of a rectangle whose area is:

a) 64 sq. cm
b) 144 sq. cm
c) 100 sq. cm
d) 196 sq.cm
e) 24 sq. cm
f) 48 sq. cm
g) 96 sq. cm
h) 72 sq. cm

If the class has fewer than eight groups, assign some groups more than one rectangle.

Ask each group to replace the DATA in line 90 of the presented program with their derived pairs of integers. Then ask the students to run the program and examine the data in order to determine the rectangle with the smallest perimeter. Ask the students to conclude that the rectangle whose dimensions are closest to that of a square has the smallest perimeter.

vii) A similar development can be used in conjunction with the program that follows. Here the students are asked to determine all integers that can be used as the measures of the dimensions of a rectangle whose perimeter is 24 cm. The program is then used to determine which rectangle has the largest area.

Program

```
10  PRINT "COMPARING THE AREAS OF RECTANGLES"
20  PRINT "LENGTHS",    "WIDTHS"
25  PRINT "AREAS"
30  PRINT "(IN CM)", "(IN CM)", "(IN CM)"
40  READ L,W
50  IF L=0 THEN 120
60  A=L*W
70  PRINT L,W,A
80  GOTO 40
90  DATA 11,1,10,2,9,3
100 DATA 8,4,7,5,6,6
110 DATA 0,0
120 END
```

viii) A similar type of program structure can be used to compile data to be examined from a graphic viewpoint. For example, suppose the program below is used to compile data on a flu epidemic where the value calculated and stored in memory cell P will represent the healthy population for the given number of days stored in memory cell D. The students can be given a program as follows.

Program

```
10  PRINT "EXAMINING A FLU EPIDEMIC"
20  PRINT "HEALTHY POP.", "# OF DAYS"
30  READ D
40  IF D=-99 THEN 110
50  P=100000-60*D*D+D*D*D
60  PRINT P,D
70  GOTO 30
80  DATA 0,4,8,12,16,20,24,28
90  DATA 32,36,40,44,48,52,56,60
100 DATA -99
110 END
```

A sample printout for this program is shown below.

```
RUN
EXAMINING A FLU EPIDEMIC
HEALTHY POP.    # OF DAYS
100000          0
99104           4
96672           8
93088           12
88736           16
84000           20
79264           24
74912           28
71328           32
68896           36
68000           40
69024           44
72352           48
78368           52
87456           56
100000          60
```

Ask the students to vary the numeric intervals listed in the DATA statements in lines 80 and 90 so that they will have enough data to produce a useful graph. Ask the students to exam-

ine the data and graph in order to answer the following questions.

- For how many days does the healthy population continue to decrease?
- For how many days does the decrease in the healthy population proceed at an increasing rate?
- For which days is the decrease in the healthy population proceeding at a decreasing rate?
- On which day is the healthy population the smallest?
- What is the approximate healthy population on the day that it is the smallest?
- On which day does the healthy population begin to increase?

Note: For lessons similar to Lesson C, you may wish to prepare the programs on a disk in advance so that it will not be necessary for the students to type the programs into the computer.

Lesson D

This lesson serves as a model to illustrate how computers can be used as tools in probability experiments.

i) The students should be able to distinguish between empirical and theoretical probability. They should be familiar with the appropriate vocabulary; e.g., outcome, event, sample space, accumulated, equally likely, biased, unbiased, probability ratio, etc. They should be able to determine the theoretical probability ratio of an event's occurring.

ii) Divide the students into groups of four. Ask each group to conduct one of the three simulations by running the programs listed below. Provide the groups with recording sheets and calculators.

Simulation I

Instructions:

1. Each student is to run once a program that simulates rolling a die fifty times and is to record the number of times that a "2" is rolled.

   Program

```
10 PRINT "ROLLING A 2"
20 PRINT "WITH A SINGLE DIE"
30 C=0
40 FOR T=1 TO 50
50 D=INT(6*RND(1))+1
60 IF D=2 THEN C=C+1
70 NEXT T
80 PRINT "A 2 WAS ROLLED"; C; "TIMES."
90 END
```

   Note: The random number function (RND(1)) as used in line 50 of this program is specifically for the Apple computer. Check your manual for the appropriate adjustment for use with other computers.

2. The data is to be recorded on a recording sheet similar to the one that follows.

Students' Names	Number of Times a "2" was Rolled	Number of Rolls	Accumulated Number of Times a "2" was Rolled (A)	Accumulated Number of Rolls (B)	Experimental Probability $\left(\frac{A}{B}\right)$
		50		50	
		50		100	
		50		150	
		50		200	

3. The student should compute and enter in column (A) the accumulated number of times a "2" was rolled. Then the ratio of column (A) to column (B) is the experimental probability and should be entered in column (A/B) as a three-place decimal.

Simulation II

Instructions:

1. Each student is to run once a program that simulates tossing a coin fifty times and is to record the number of times a "head" is tossed.

   Program

```
10 PRINT "TOSSING A HEAD"
20 PRINT "WITH A SINGLE COIN"
30 C=0
40 FOR T=1 TO 50
50 D=INT(2*RND(1))+1
60 IF D=1 THEN C=C+1
70 NEXT T
80 PRINT "A 'HEAD' WAS TOSSED"; C; "TIMES."
90 END
```

2. The data is to be recorded on a recording sheet similar to the one that follows.

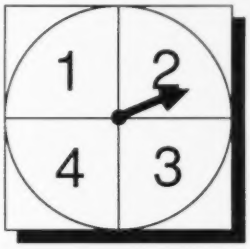
Students' Names	Number of Times a "Head" was Tossed	Number of Tosses	Accumulated Number of Times a "Head" was Tossed (A)	Accumulated Number of Tosses (B)	Experimental Probability $\left(\frac{A}{B}\right)$
		50		50	
		50		100	
		50		150	
		50		200	

3. The student should compute and enter in column (A) the accumulated number of times a "head" was tossed. Then, the ratio of column (A) to column (B) is the experimental probability and should be entered in column (A/B) as a three-place decimal.

Simulation III

Instructions:

1. Each student is to run once a program that simulates spinning a spinner fifty times and is to record the number of times that a "3" appears.



Program

```
10  PRINT "A 3 APPEARING"
    20  PRINT "ON A SINGLE SPINNER"
    30  C=0
    40  FOR T=1 TO 50
    50  D=INT(4*RND(1))+1
    60  IF D=3 THEN C=C+1
    70  NEXT T
    80  PRINT "A '3' APPEARED"; C; "TIMES."
    90  END
```

2. The data is to be recorded on a recording sheet similar to the one that follows.

Students' Names	Number of Times a "3" Appeared	Number of Spins	Accumulated Number of Times a "3" Appeared (A)	Accumulated Number of Spins (B)	Experimental Probability $\left(\frac{A}{B}\right)$
		50		50	
		50		100	
		50		150	
		50		200	

3. The students should compute and enter in column (A) the accumulated number of times that a "3" appeared. Then, the ratio of column (A) to column (B) is the experimental probability and should be entered in column (A/B) as a three-place decimal.

iii) Ask the students to compare the empirical and theoretical probability. Ask them if each simulation provides evidence that the theoretical probability assigned is correct. Ask the students to notice that as the accumulated number of rolls, tosses, or spins increases, the empirical probability comes closer to the theoretical probability.

iv) You may wish to have the students do these simulations as experiments using real dice, coins, and spinners.

Conclusion

In summary, this discussion is intended to serve as a resource and guide for the classroom teacher in the long-term planning of computer instruction in *General High School Mathematics.*

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• THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

Introduction

This section is designed to present a model for the infusion of an ongoing component of computer instruction into the *Three-Year Sequence For High School Mathematics.* This model is designed to review, reinforce, and extend the content of these secondary school curricula. As students' interest in computers is directed toward mathematics, they not only become computer literate but also better students of mathematics.

In this context, computer literacy is defined as how a computer can be used to solve problems. However, problem solving in this model is not entirely the same process as in the traditional mathematics course. It is an activity that addresses the very foundation of the cognitive skills that are required in mathematical problem solving. The student who tutors a fellow student can be used as an analogy here. The tutor, when forced to explain concepts and ideas, crystallizes his or her thoughts in the process; thus he or she develops a clearer understanding of the topic. Similarly, when the student programs a computer, he or she must break a problem down into component parts in order to translate it into a language understandable to a computer.

One of the essayists in *Bridges to Infinity: The Human Side of Mathematics* observed that unlike the scientist who deals with what is, the mathematician is concerned with what can be. The use of the computer as a problem-solving tool in mathematics will not only provide the students a genuine comprehension of the topics in the *Three-Year Mathematics Sequence* but, concomitantly, give them a vehicle for the exploration of mathematical ideas.

Students will begin Course I of *Sequential Mathematics* with varied preparation in computer programming in BASIC. Some may have a rudimentary knowledge acquired before they reach the secondary school or in a concurrent computer science course at the secondary level. Others may have little or no knowledge, while still others may be quite proficient. Peer tutoring, programming texts, and other resource materials should be made available to the students either in the classroom, in the mathematics library, or in the school library media center.

Occasionally, the knowledge of computer programming may have to be supplemented with brief lessons or with teacher-prepared materials designed to teach a specific computer idea; for example, rounding numbers, the random number function, subroutines, or subscripted variables may have to be reviewed or taught. The use of the logical operators parallels the *Three-Year Sequence*.

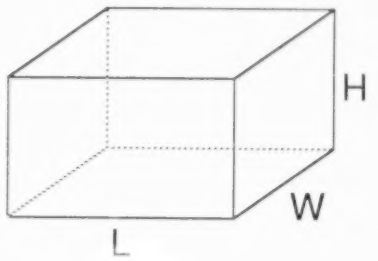
Microcomputers should be available for students to use during class time, when appropriate, both in a laboratory setting and for classroom demonstration. Computers should also be available at other times of the day for the students' use. This can be accomplished by opening the computer laboratory before and/or after the school day as well as throughout the entire school day. Computer reference manuals should be made available for the students' use.

Some examples of specific problems that can be used for computer solutions in each course of *Sequential Mathematics* follow. For each problem, an analysis, a plan (and/or flowchart) and a BASIC program are provided. The problems are meant to be suggestive, not prescriptive. The analyses and plans may be integrated into the traditional lessons dealing with these topics. The plans may also be given for homework. The programs and sample runs could be done after class, as projects, or as homework.

Generally, the analysis can be derived from the students by asking, "How do we ...?" The plans or flowcharts can be obtained by asking, "How can we use the algorithm in order to solve the problem?" If students have received instruction in the preparation of flowcharts, you may require flowcharts as components of the solutions.

Course I

Topic: Surface Areas and Volumes of Rectangular Solids



Example:

Enter the length, width, and height of a rectangular solid. Have the computer find and display the surface area and volume.

Analysis: The surface of a rectangular solid consists of six rectangles whose areas are given as follows.

Area of the front = LH
Area of the back = LH
Area of the bottom = LW
Area of the top = LW
Area of the left side = HW
Area of the right side = HW

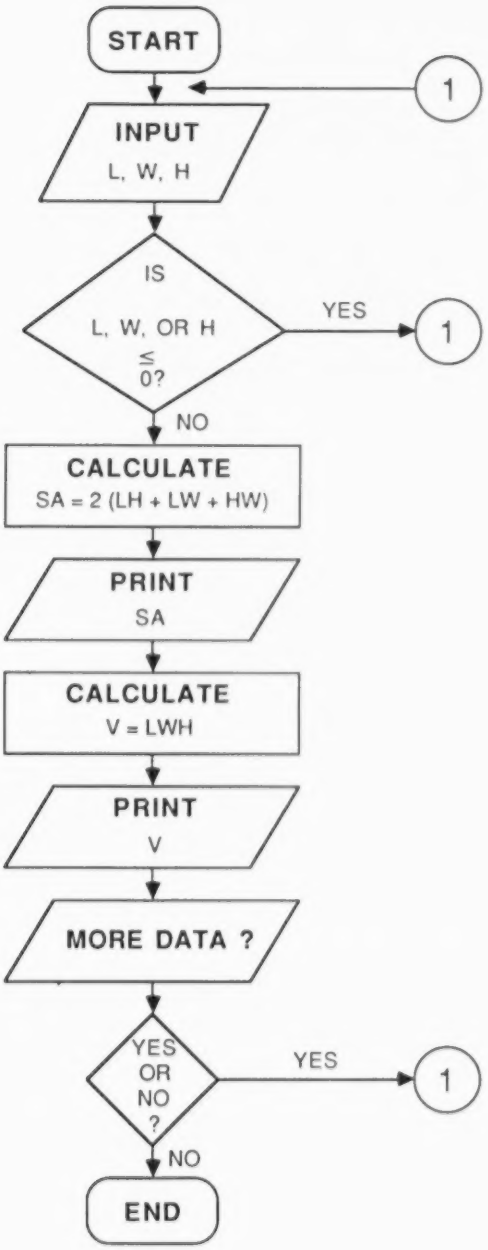
The students may conclude that the surface area of a rectangular solid may be computed by using any one of the following formulas:

$SA = LH+LH+LW+LW+HW+HW$
$SA = 2LH+2LW+2HW$
$SA = 2(LH+LW+HW)$.

The volume of the same figure may be found using the formula $V = LWH$.

Plan:

1) Enter values for the length, width, and height of a rectangular solid: L, W, H.
2) Check: Is L, W, or H less than or equal to zero?
   a) If yes, then go back to 1.
   b) If no, then continue.
3) Calculate the surface area: $SA = 2(LH+LW+HW)$.
4) Print the surface area, SA.
5) Calculate the volume: $V = LWH$.
6) Print the volume, V.
7) Check: Do you wish to enter more data?
   a) If yes, then go back to 1.
   b) If no, then continue.
8) End.



Program:

```
10  PRINT "SURFACE AREA AND VOLUME OF"
20  PRINT "*** A RECTANGULAR SOLID ***"
30  PRINT
40  PRINT "ENTER THE LENGTH, WIDTH, AND
    HEIGHT."
50  INPUT L, W, H
60  IF L <=0 OR W <=0 or H <=0 THEN PRINT
    "MEASURES MUST HAVE POSITIVE VALUES."
    :GOTO 30
70  SA=2*(L*H+L*W+H*W)
80  V=L*W*H
90  PRINT "SURFACE AREA IS"; SA; "SQUARE
    UNITS."
100 PRINT "VOLUME IS"; V; "CUBIC UNITS."
110 PRINT
120 PRINT "DO YOU HAVE MORE DATA?"
130 INPUT "ANSWER YES OR NO"; R$
140 IF R$="YES" THEN 30
150 PRINT "THE END"
160 END
```

Related Problems:

1. Enter the length of the edge of a cube. Have the computer find and display the volume and surface area.
2. Write programs that can be used to answer the following questions.
   a) If the measure of the edge of a cube is doubled, what happens to the surface area? to the volume?
   b) If the length, width, and height of a rectangular solid are each increased by 20%, by what percent is the volume of the rectangular solid increased?

Topic: Pythagorean Theorem

Example:

Enter the lengths of the legs of a right triangle. Have the computer find the length of the hypotenuse rounded to the nearest tenth.

Analysis: To find the length of the hypotenuse, we use the formula $c = \sqrt{a^2 + b^2}$ where a and b represent the lengths of the legs and c represents the length of the hypotenuse. Checks should be made to ensure that the entered lengths are positive.

Plan:

1) Enter the lengths of the legs of a right triangle: A, B.
2) Check: Is A or B <=0?
   a) If yes, then go to 5.
   b) If no, then continue.
3) Calculate the length of the hypotenuse: $C = \sqrt{A^2 + B^2}$.
4) Round the value in memory cell C to the nearest tenth.
5) Check: Do you wish to enter more data?
   a) If yes, then go back to 1.
   b) If no, continue.
6) End.

Program:

```
10  PRINT "THE PYTHAGOREAN THEOREM"
20  INPUT "ENTER THE LENGTHS OF THE LEGS";
    A, B
30  IF A <=0 OR B <=0 THEN 120
40  C=SQR(A^2+B^2)
50  CR=INT(10*(C+.05))/10
60  PRINT "THE HYPOTENUSE IS"; CR
70  PRINT:PRINT:PRINT
80  PRINT "DO YOU HAVE MORE DATA?"
90  INPUT "ANSWER YES OR NO"; R$
100 IF R$="YES" THEN 20
110 GOTO 140
120 PRINT "THE LENGTHS CANNOT BE NONPOSI-
    TIVE."
130 GOTO 80
140 END
```

Related Problems:

1. Enter the lengths of a leg and the hypotenuse of a right triangle. Have the computer find and display the length of the other leg rounded to the nearest hundredth. (How will the computer "know" which length is that of the hypotenuse?)
2. Enter three numbers. Have the computer determine if the numbers can represent the lengths of the sides of a right triangle. If it is a right triangle, have the computer find and display its area and perimeter; otherwise, display, NOT A RIGHT TRIANGLE.
3. Enter the measures of two angles of a triangle. Have the computer determine and display whether the triangle is acute, obtuse, or right.

Topic: Solving Systems of Linear Equations in Two Variables

Example:

Have the computer solve a system of linear equations in two variables.

Analysis: The answer to the question "How do we solve a system of equations in two variables?" requires the generalization of the procedure that the students have been using to solve such equations. If we begin with the equations

$$AX+BY=C$$
$$DX+EY=F,$$

where X and Y are the variables and A, B, C, D, E, and F are constants, we solve for X and Y as follows.

To eliminate the variable Y:

1) Multiply both sides of the first equation by E:
$$E(AX+BY) = E(C)$$
which becomes $AEX+BEY = CE$.
2) Multiply both sides of the second equation by $-B$:
$$-B(DX+EY) = -B(F)$$
which becomes $-BDX-BEY = -BF$.
3) Adding the two equations, we get:
$$AEX-BDX = CE-BF.$$
4) Factoring gives us:
$$(AE-BD)X = CE-BF.$$
5) By dividing by the coefficient of X, our result is:
$$X = \frac{CE-BF}{AE-BD}.$$

To eliminate the variable X:

1) We multiply the first equation by $-D$:
$$-D(AX+BY) = -D(C)$$
which becomes $-ADX-BDY = -CD$.
2) We multiply the second equation by A:
$$A(DX+EY) = A(F)$$
which becomes $ADX+AEY = AF$.
3) Adding the two equations, we get:
$$AEY-BDY = AF-CD.$$
4) Factoring gives us:
$$(AE-BD)Y = AF-CD.$$
5) By dividing by the coefficient of Y, our result is:
$$Y = \frac{AF-CD}{AE-BD}.$$

The general solution is $X = \frac{CE-BF}{AE-BD}$ and $Y = \frac{AF-CD}{AE-BD}$, known as Cramer's Rule. (You might introduce 2x2 determinants as an enrichment activity at this point.)

Since computing the values of X and Y both require division by the value AE-BD, we must be sure that value is not equal to zero.

Plan:

1) Enter the values of the constants: A, B, C, D, E, F.
2) Check: Is $AE-BD = 0$?
   a) If yes, then print NO UNIQUE SOLUTION. Then go back to 1.
   b) If no, then continue.
3) Calculate the value of X using Cramer's Rule:
$$X = \frac{CE-BF}{AE-BD}.$$
4) Calculate the value of Y using Cramer's Rule:
$$Y = \frac{AF-CD}{AE-BD}.$$
5) Print the values of X and Y.
6) Check: Do you wish to enter more data?
   a) If yes, then go back to 1.
   b) If no, then continue.
7) End.

Program:

```
10  PRINT "SOLVING SIMULTANEOUS EQUATIONS"
20  PRINT
30  INPUT "FOR AX+BY=C, ENTER A, B, C"; A, B, C
40  INPUT "FOR DX+EY=F, ENTER D, E, F"; D, E, F
50  IF A*E-B*D=0 THEN 170
60  X=(C*E-B*F)/(A*E-B*D)
70  Y=(A*F-C*D)/(A*E-B*D)
80  PRINT "THE SOLUTION TO"
90  PRINT A; "X+"; B; "Y=";C
100 PRINT D; "X+"; E; "Y=";F
110 PRINT "IS ("; X;","; Y; ")"
120 PRINT:PRINT:PRINT
130 PRINT "DO YOU HAVE MORE DATA?"
140 INPUT "ANSWER YES OR NO";R$
150 IF R$="YES" THEN 20
160 GOTO 190
170 PRINT "NO UNIQUE SOLUTION."
180 GOTO 20
190 PRINT "ACTIVITY TERMINATED!"
200 END
```

Related Problem:

Determine if AX+BY = C and DX+EY = F, where X and Y are variables and A, B, C, D, E, and F are constants, are the equations of lines which are parallel, coincident, or intersecting. If the lines intersect, find and display the point of intersection.

Course II

Topic: Solving Quadratic Equations

Example:

Solve for X all equations of the form $AX^2+BX+C=0$.

Analysis: To solve quadratic equations, we can use the formula, $X = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$.

We need to check two things with regard to the quadratic formula:

1) We must check if $A = 0$.
2) We must check if $B^2-4AC < 0$.

Plan:

1) Enter the values of the constants: A, B, C.
2) Check: Is A = 0?
   a) If yes, then go back to 1.
   b) If no, then continue.
3) Check: Is $B^2-4AC < 0$?
   a) If yes, then print ROOTS ARE IMAGINARY. Then go back to 1.
   b) If no, then continue.
4) Calculate the roots:

$$R1 = \frac{-B+\sqrt{B^2-4AC}}{2A} \text{ and } R2 = \frac{-B-\sqrt{B^2-4AC}}{2A}$$

5) Print the values of the roots, R1 and R2.
6) Check: Do you wish to enter more data?
   a) If yes, then go back to 1.
   b) If no, then continue.
7) End.

Program:

```
10  PRINT "SOLVING QUADRATIC EQUATIONS"
20  PRINT "FOR AX^2+BX+C,"
30  INPUT "WHAT ARE THE VALUES OF A, B, AND C"; A, B, C
40  IF A=0 THEN GOTO 150
50  IF B*B-4*A*C <0 THEN GOTO 160
60  PRINT "THE ROOTS ARE"
70  R1=(-1*B+SQR(B*B-4*A*C))/(2*A)
80  R2=(-1*B-SQR(B*B-4*A*C))/(2*A)
90  PRINT "X="; R1; AND "X="; R2
100 PRINT:PRINT
110 PRINT "DO YOU HAVE MORE DATA?"
120 INPUT "ANSWER YES OR NO"; R$
130 IF R$="YES" THEN PRINT: GOTO 20
140 GOTO 170
150 PRINT "'A' CANNOT BE ZERO.": GOTO 20
160 PRINT "THE ROOTS ARE IMAGINARY.": PRINT: GOTO 20
170 END
```

Related Problems:

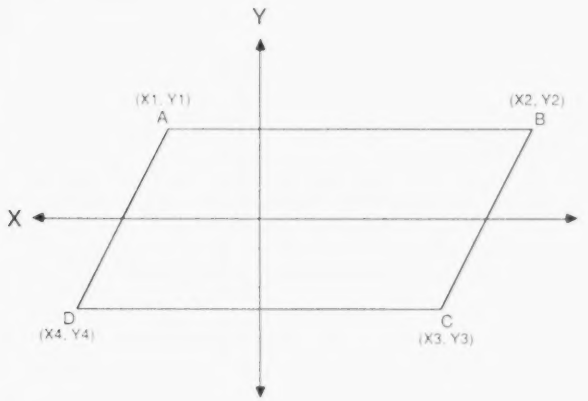
1. Determine if the roots of a quadratic equation are...
   i) real or imaginary;
   ii) rational or irrational;
   iii) equal or unequal.
2. Given A, B, and C to represent the coefficients from the general formula of a parabola $Y = AX^2+BX+C$ where $A \neq 0$, have the computer print...
   i) the coordinates of the point or points of intersection of the graph with the x-axis;
   ii) the equation of the axis of symmetry;
   iii) the coordinates of the turning point, stating whether the point is a maximum or a minimum.

Topic: "Proof" by Analytic Geometric Means

Example:

Enter the coordinates of a quadrilateral ABCD. Have the computer determine if the quadrilateral is a parallelogram.

Analysis: One way to show that a quadrilateral is a parallelogram is to show that its opposite sides are congruent. We must therefore use the distance formula to find the length of each side and then compare the lengths of the opposite sides.



Plan:

1) Enter the coordinates of the quadrilateral: A, B, C, D.
2) Calculate the lengths of the sides:

   $AB = \sqrt{(X2-X1)^2 + (Y2-Y1)^2}$

   $BC = \sqrt{(X3-X2)^2 + (Y3-Y2)^2}$

   $DC = \sqrt{(X4-X3)^2 + (Y4-Y3)^2}$

   $AD = \sqrt{(X4-X1)^2 + (Y4-Y1)^2}$.

3) Check: Is AB = DC and BC = AD?
   a) If yes, then print PARALLELOGRAM. Then go to 5.
   b) If no, then continue.
4) Print NOT A PARALLELOGRAM.
5) Check: Do you wish to enter more data?
   a) If yes, then go back to 1.
   b) If no, then continue.
6) End.

Program:

```
10  PRINT "IS THE QUADRILATERAL A PARALLELOGRAM?"
20  PRINT:PRINT "ENTER THE COORDINATES OF"
30  PRINT "QUADRILATERAL ABCD"
40  INPUT "A (X1,Y1)"; X1, Y1
50  INPUT "B (X2,Y2)"; X2, Y2
60  INPUT "C (X3,Y3)"; X3, Y3
70  INPUT "D (X4,Y4)"; X4, Y4
80  AB=SQR ((X2-X1)^2+(Y2-Y1) ^2)
90  BC=SQR ((X3-X2)^2+(Y3-Y2) ^2)
100 DC=SQR ((X4-X3)^2+(Y4-Y3) ^2)
110 AD=SQR ((X4-X1)^2+(Y4-Y1) ^2)
120 IF AB=DC and BC=AD THEN 180
130 PRINT "IT'S NOT A PARALLELOGRAM."
140 PRINT:PRINT "DO YOU HAVE MORE DATA?"
150 INPUT "ANSWER YES OR NO"; R$
160 IF R$="YES" THEN GOTO 20
170 GOTO 200
180 PRINT "IT'S A PARALLELOGRAM!"
190 GOTO 140
200 END
```

Related Problems:

1. Enter the coordinates of a quadrilateral ABCD. Have the computer determine if the quadrilateral is a rectangle and if so, print its area and perimeter.
2. Enter the coordinates of two points. Have the computer find and display the slope of the line passing through the points.
3. Enter the coordinates of two points. Have the computer print the equation of the line determined by the points.
4. Enter the coordinates of a triangle. Have the computer classify the triangle as equilateral, isosceles, or scalene.
5. Enter the coordinates of a quadrilateral ABCD. Have the computer determine if the quadrilateral is a rhombus and if so, print its area and perimeter.
6. Enter the coordinates of a triangle. Have the computer classify the triangle as acute, right, or obtuse.

   Note: Before assigning this problem, it is recommended that a discussion regarding the extension of the Pythagorean Theorem take place in class; e.g., for a triangle whose sides have lengths, represented by A, B, and C, if:

   i) $A^2 + B^2 > C^2$, then the triangle is acute;
   ii) $A^2 + B^2 = C^2$, then the triangle is right;
   iii) $A^2 + B^2 < C^2$, then the triangle is obtuse.

Topic: Permutations and Combinations

Example:

Have the computer find the value of all possible permutations of n objects taken r at a time (${}_nP_r$, where $n \geq r$ for whole numbers n and r).

Analysis:

a) In order to use the formula ${}_nP_r = \frac{n!}{(n-r)!}$ we must have the computer determine n! and (n−r)!. Rather than write separate statements for computing the factorials, we can insert a single program segment into the main program as a subroutine.

b) We must include provisions in our program for 0! = 1.

Plan for the Main Program:

1) Enter the values of N and R.
2) In order to save the original value input for N, we assign the initial value to memory cell A and will use memory cell A in our subroutine: A = N.
3) Send the value in memory cell A into a subroutine in order to determine N! When the computer returns to the main program, the value of N! will be stored in memory cell B.
4) In order to save the value of N!, we assign its value, which is stored in memory cell B, to memory cell X: X = B.
5) In order to have the computer determine (N−R)!, we must send the value of N−R into the subroutine and in order to send the value of N−R into the subroutine, we must store it in memory cell A: A = N−R. When the computer returns to the main program, the value of (N−R)! will be stored in memory cell B.
6) We assign the value of (N−R)!, which is stored in memory cell B, to memory cell Y: Y = B.
7) Calculate the required quotient of the factorials: Q = X/Y.
8) Print the value of ${}_nP_r$: Q.
9) Check: Do you wish to enter more data?
   a) If yes, then go back to 1.
   b) If no, then continue.
10) End.

Plan for the Subroutine:

1) Each time control transfers to the subroutine, the number whose factorial we want the computer to calculate is stored in memory cell A. When control transfers back to the main program, the value of the factorial is stored in memory cell B.
2) Initialize the value of the product leading to the factorial being sought: P = 1.
3) Check: Is A = 0?
   a) If yes, then go to 4a.
   b) If no, then continue.
4) Check: Is A = 1?
   a) If yes, then assign to memory cell B the value in P. Then return to the main program.
   b) If no, then continue.
5) Calculate the next product: P = P•A.
6) Decrease the value in memory cell A by one: A = A−1.
7) Go back to 4 to determine if the computer can stop finding the product, store the value of the factorial in memory cell B, and then return to the main program.

Program:

```
10  PRINT "FINDING THE NUMBER OF PERMUTATIONS"
20  PRINT "OF N OBJECTS TAKEN R AT A TIME"
30  PRINT:PRINT
40  INPUT "N="; N
50  INPUT "R="; R
60  A=N
70  GOSUB 500
80  X=B
90  A=N-R
100 GOSUB 500
110 Y=B
120 Q=X/Y
130 PRINT "THE NUMBER OF PERMUTATIONS IS"; Q: PRINT
140 PRINT "DO YOU HAVE MORE DATA?"
150 INPUT "ANSWER YES OR NO"; R$
160 IF R$= "YES" THEN 30
170 GOTO 580
500 P=1
510 IF A=0 THEN 560
520 IF A=1 THEN 560
530 P=P*A
540 A=A-1
550 GOTO 520
560 B=P
570 RETURN
580 END
```

Related Problems:

1. Have the computer determine the value of ${}_nP_r$, using the formula ${}_nP_r = n(n-1)(n-2)...(n-r+1)$.
2. Have the computer find the value of all possible combinations of n objects taken r at a time (${}_nC_r$ where $n \geq r$ for whole numbers n and r).

Course III

Topic: Mean and Standard Deviation

Example:

Enter the number of data, N. Have the computer find and display the mean and the standard deviation.

Analysis: We will use the formulas:

$$M = \frac{\sum_{I=1}^{N} X_I}{N} \quad \text{and} \quad SD = \sqrt{\frac{1}{N}\sum_{I=1}^{N}(M - X_I)^2}$$

where M = the mean,
N = the number of data,
X_I = each item of the data,
I = number of the position or order of each X_I in the list of data, and
SD = the standard deviation.

Plan:

1) Enter the number of data, N.
2) Set aside additional memory space for the array, X(I), which will be used for all the data: DIM X(N).
3) Initialize a memory cell which will be used to sum (accumulate) the data: S=0.
4) Set up a FOR/NEXT loop to count out the position of the data in the list: FOR I = 1 TO N.
5) Enter each item of the data, X(I).
6) Add to the sum already stored in memory cell S: S = S + X(I).
7) NEXT I.
8) Calculate the mean: M = S/N.
9) Reinitialize memory cell S at zero so that it can be used to store the sums of the squares of $(M-X_I)$ for $1 \le I \le N$: S = 0.
10) Again, set up a FOR/NEXT loop to count out the position of the data in the list: FOR I = 1 TO N.
11) Add to the sum already stored in memory cell S: S = S + (M−X(I))^2.
12) NEXT I.
13) Calculate the standard deviation: SD = SQR(S/N).
14) Display M, SD.
15) Check: Do you wish to enter another list of data?
    a) If yes, then go back to 1.
    b) If no, then continue.
16) End.

Program:

```
10  PRINT "FINDING THE MEAN AND THE STANDARD DEVIATION"
20  PRINT:PRINT
30  INPUT "HOW MUCH DATA";N
40  PRINT "ENTER THE DATA."
50  DIM X(I)
60  S=0
70  FOR I=1 TO N
80  INPUT X(I)
90  S=S+X(I)
100 NEXT I
110 M=S/N
120 S=0
130 FOR I=1 TO N
140 S=S+(M-X(I))^2
150 NEXT I
160 SD=SQR(S/N)
170 PRINT "THE MEAN IS";M
180 PRINT "THE STANDARD DEVIATION IS"; SD
190 PRINT
200 PRINT "DO YOU HAVE MORE DATA?"
210 INPUT "ANSWER YES OR NO"; R$
220 IF R$="YES" THEN 20
230 END
```

Related Problems:

1. A shortcut formula can be used to compute the standard deviation:

$$SD = \sqrt{\frac{N\sum_{I=1}^{N} X_I^2 - \left(\sum_{I=1}^{N} X_I\right)^2}{N^2}}$$

where SD = the standard deviation,
N = the number of data,
X_I = each item of the data, and
I = the number of the position or order of each X_I in the list.

2. Enter the height for each student in your mathematics class. Have the computer calculate the mean height and the standard deviation.
3. Take any chart from an almanac. Have the computer find the mean, median, mode, and standard deviation for the data contained in the chart. (Students familiar with a computer algorithm for sorting can also be asked to have the computer find the median and the mode.)

Topic: Trigonometric Identities and Equations

Example:

Verify the trigonometric identity $\tan X = \frac{\sin X}{\cos X}$ for any angle X measured in degrees.

Analysis: We will compare tan X to $\frac{\sin X}{\cos X}$ for each number of degrees, X, entered. For the comparison, we will round off the values to four decimal places. Also, in order to have the computer determine tan X, sin X, and cos X, we must change the degrees to radians.

Plan:

1) Enter the measure of the angle in degrees, X.
2) Convert the degrees to radians: R = X(π/180) where π = 3.1416.
3) Calculate the quotient: Q = sin R/cos R.

4) Calculate the tangent of angle X:T = tan R.
5) Round off Q and T to four decimal places.
6) Check: Is Q = T?
   a) If yes, then print IDENTITY IS TRUE. Then go to 8.
   b) If no, then continue.
7) Print IDENTITY IS NOT TRUE.
8) Check: Do you wish to enter more data?
   a) If yes, then go back to 1.
   b) If no, then continue.
9) End.

Program:

```
10  PRINT "VERIFYING TAN X=SIN X/COS X"
20  INPUT "HOW MANY DEGREES";X
30  R=X*3.1416/180
40  Q=SIN(R)/COS(R)
50  T=TAN(R)
60  Q=Q+.00005: T=T+.00005
70  Q=INT(10000*Q)/10000
80  T=INT(10000*T)/10000
90  PRINT "SIN"; X;"/COS"; X;"="; Q
100 PRINT "TAN"; X;"="; T
110 IF T=Q THEN GOTO 140
120 PRINT "IT IS NOT AN IDENTITY."
130 GOTO 160
140 PRINT "IDENTITY VERIFIED FOR"; X; "DE-
    GREES."
150 PRINT
160 PRINT "DO YOU WISH TO ENTER MORE
    DATA?"
170 INPUT "ANSWER YES OR NO"; R$
180 IF R$="YES" THEN 20
190 END
```

Related Problems:

1. Enter the integral measure of an angle between 0° and 360°. Have the computer express the sine of the angle as a function of a positive acute angle.
2. Verify the identity $\sin^2 x + \cos^2 x = 1$ for all integral values of x from 0 degrees through 90 degrees.
3. Solve $A\tan^2 x + B\tan x + C = 0$ for tan x. For each value of tan x, have the computer display the quadrant numbers which contain the values of x which solve the problem.

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SECTION III

Computer Mathematics – A Second Year of Mathematics for a Local Diploma

THE REGENTS ACTION PLAN

The New York State Regents Action Plan requires the successful completion of two years of mathematics for a diploma. A Regents diploma requires passing Courses I and II of the *Three-Year Sequence for High School Mathematics* as well as the respective State Regents examination in each course. For a local diploma, several options are available to meet this two-year requirement, one of which may be computer mathematics. Several schools and school districts have implemented computer mathematics courses that have been approved by the New York State Education Department for credit as second-year mathematics courses.

PURPOSE

The purpose of Computer Mathematics is to motivate and prepare students to continue their study of mathematics. The motivation will come by directing student interest in computers toward mathematics. The preparation will come by using a metacognitive approach to solving problems. That is, students will be taught how to think about solving problems. This stress on thinking skills will enable students to transfer newly acquired strategies to other problem-solving situations more easily.

Developing computer solutions to problems requires higher level thinking skills. In this course, a generalized solution is being formalized. Generalizing is usually difficult for students who are not succeeding in traditional mathematics courses. However, the computer enables them to develop a connection between a concrete example and the abstract generalization. This kind of activity will enhance students' understanding of the conceptual background and will guide them to succeed in more advanced mathematics courses.

COURSE OUTLINE

The following suggested list represents a collection of topics that were included in course outlines for computer mathematics approved by the State Education Department. This listing is not exhaustive but merely presents successfully fieldtested topics. A sample lesson plan will be provided to highlight a questioning technique that may be used to illustrate the metacognitive approach that is being recommended. Suggestions for implementing this model with a focus on scheduling students for classroom and laboratory lessons and evaluating student performance will also be explored.

A. Using beginning programming concepts
   1. Finding perimeters of polygons
      a. INPUT
      b. Numeric variables
      c. PRINT
      d. Arithmetic operation symbols (+,–,*,/)
      e. Calculations
      f. END
   2. Finding areas of quadrilaterals and triangles
      a. Printing text
      b. Skipping lines
      c. REM
      d. String variable
   3. Converting binary numerals into decimal numerals
   4. Discounts, sale price, and final cost
   5. Finding the arithmetic mean
   6. Finding simple and compound interest
   7. Getting paid – hourly wage and straight commission
   8. Finding supplements and complements of angles
   9. Finding the sum of the angles of a triangle
   10. Solving, for x, equations of the form $x+a=b$, $x-a=b$, $ax=b$, $x/a=b$
   11. Converting Celsius and Fahrenheit temperatures
   12. Reviewing areas with metric conversions

B. Using intermediate programming concepts
   1. Solving, for x, equations of the form $ax+b=c$ and $ax-b=c$
      a. IF – THEN
      b. GOTO

2. Classifying triangles by angles and sides
3. Discovering divisibility rules
   a. An alternate form of the INPUT statement
   b. INT(X)
4. Using the formula D=RT
   a. READ – DATA
   b. Counting
   c. TAB(X)
5. Finding prime numbers
   a. FOR – NEXT loop
   b. TAB(X)
6. Finding factors of numbers
   a. An accumulator
   b. Multiple statements on a line
7. Determining if three numbers can represent the lengths of the sides of a triangle
   a. Logical operators
8. Finding the circumference and area of a circle
   a. SQR(X)
   b. Rounding
9. Pythagorean Theorem
10. Pythagorean Triples
11. Classifying triangles by angles when given the measures of the lengths of their sides
12. Fibonacci sequence

C. Using advanced computer programming concepts
1. Developing an inventory program
   a. Subscripted numeric variable
   b. Subscripted string variable
2. Writing simulations
   a. RND(X)
   b. Selecting parameters for random numbers
3. Using string functions
   a. MID$
   b. LEN
4. Finding the median
   a. Sorting numbers
   b. Sorting strings
5. Finding the area under a curve

SAMPLE OF A CLASSROOM LESSON PLAN

Aim: How do we determine the discount and the sale price of an item?

Performance Objectives:

The student will be able to:

1. find the percent of a number;
2. find the amount of discount;
3. find the sale price;
4. demonstrate his/her understanding of finding the discount and sale price by developing a plan for a computer solution to a related problem.

Vocabulary: discount, sale price, regular (original, list, marked) price, rate of discount

Motivation:

Mrs. Sanders received 12% off the price of a computer that regularly sells for $450. How much did Mrs. Sanders pay for the computer?

Development:

1a. While reviewing the motivation problem with the students, ask them to identify the regular price, the rate of discount, the discount, and the sale price. Stress the difference between the rate of discount and the amount of discount.

1b. Ask the students to solve the motivation problem as follows.

i) Change 12% to a decimal. Remind the students that percent means "per hundred"; e.g., 12% means 12 per hundred or 12/100. Since the bar means "divided by," to change a percent to a decimal, we must divide by 100. (Students will use this information when developing computer algorithms to solve problems involving percent.) When we divide by 100, we move the decimal point two places to the left.

ii) To determine the amount of discount, we must find .12 of $450 as shown below.

Discount = Rate of Discount x Regular Price

Discount = .12 x 450

Discount = $54

Mrs. Sanders saved $54 when she bought the computer.

iii) To determine the sale price, we subtract the discount from the regular price as follows.

Sale Price = Regular Price – Amount of Discount

Sale Price = $450 – $54

Sale Price = $396

2. Assign several application problems. An example follows.

A merchant sells a $190 suit at a 10% reduction. What is the sale price?

3. After discussing the solutions to the exercises in step 2, present the following problem to the class.

Problem: Enter the regular price of an item and the rate of discount (expressed as a decimal). Have the computer find and display the amount of discount and the sale price.

4. The students have been taught that a computer solution to a problem involves analyzing the problem, developing a plan, designing a display of the output, and coding the program.

   Analysis: The students are taught that analyzing a problem requires answering the question, "How do we solve the problem?" For the problem presented here, we answer the question, "How do we find the sale price of an item?" Since the students have already calculated the sale price, they would write the following:

   a) Given values for the rate of discount, RD, and the regular price, RP,

      Discount (D) = Rate of Discount (RD) x Regular Price (RP)

      or

      $D = RD \times RP$

   b) Sale Price(S) = Regular Price (RP) – Discount (D)

      or

      $S = RP - D$

   Plan: The students are taught that to develop a plan they must answer the question, "How do we use the algorithm to solve the problem?" For the specific problem presented here, we answer the following question, "How do we use $D = RD \times RP$ and $S = RP - D$ to solve the problem?"

   Having had many model solutions to problems in which this type of questioning was stressed, the students would write the plan below.

   1. Put in values for the regular price and the rate of discount as a decimal: RP, RD.
   2. Calculate: $D = RD \times RP$.
   3. Calculate: $S = RP - D$.
   4. Print the amount of discount, D.
   5. Print the sale price, S.
   6. End.

Summary: 1. Why do stores sell merchandise at a discount?
2. How do we determine the sale price of an item?

As part of their homework, the students might be asked to design a display and code a program to solve the problem. The next day would be a laboratory lesson. Those students who were able to write a program will have an opportunity to test their solutions on the computer. Below is a sample program that a student might write.

PROGRAM

```
10  PRINT "FINDING THE SALE PRICE OF AN ITEM"
20  PRINT "ENTER THE REGULAR PRICE OF AN ITEM."
30  INPUT RP
40  PRINT "ENTER THE RATE OF DISCOUNT AS A DECIMAL."
50  INPUT RD
60  D = RD * RP
70  S = RP - D
80  PRINT "THE DISCOUNT IS $"; D
90  PRINT "THE SALE PRICE IS $";S
100 END
```

The teacher should be available to work with those students who were unable to complete the assignment.

For students who successfully enter and run their programs, other problems should be presented to them for computer solutions. These problems should be extensions of the model presented in class. This will ensure that the transfer of learning is incremental.

The following are suggestions for supplementary problems which might be assigned during the laboratory lesson.

Problem 1: A store is having a sale giving 25% off every item. Enter the cost of an item. Have the computer print
   a. the discount
   b. the sale price.

Problem 2: Enter the regular price of an item. If the sales tax is 8%, have the computer print the final cost.

Problem 3: Enter the regular price of an item and the rate of discount as a percent. Have the computer print
   a. the discount
   b. the sale price.

SCHEDULING

Since many of the lessons in this course involve developing mathematical content, it is not necessary to schedule students to work in a computer laboratory each day. Laboratory lessons should be scheduled approximately two times a week. During the first month of the year, however, three times a week would be preferable in order to increase student familiarity with the computer.

PURPOSE OF LABORATORY LESSONS

Laboratory lessons should be used to allow students to run their own computer solutions to problems or solutions to problems that were developed by the class. Since students will complete this task at different rates, it is

necessary to provide those students who progress more rapidly with additional similar problems to solve.

EVALUATION OF STUDENT PERFORMANCE

The following criteria are suggested for grading students:

1. Classroom participation.
2. Ability of students to enter, edit, and run programs.
3. Class tests that include both mathematical content and related computer content.
4. Periodic submission of original computer solutions to teacher-assigned problems, which might include the following:
   a) statement of the problem
   b) analysis/planned solution
   c) coded program
   d) sample run of the program showing that it solves the problem.

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Computer Mathematics: An Introduction, The New York City Board of Education, Division of Curriculum and Instruction, Brooklyn, New York, 1985.

Elgarten, Gerald H. and Alfred S. Posamentier, *Using Computers: Programming and Problem Solving*, Addison-Wesley Publishing Company, Menlo Park, California, 1984.

SECTION IV

Advanced Computer Mathematics

One of the options to meet the requirements for graduation under the New York State Regents Action Plan is completion of a five-year sequence which could be mathematics. A course in advanced computer mathematics could be used for this sequence. This section will address the content and methodology of this course.

The topics to be included in advanced computer mathematics are those which are found in typical high school mathematics programs. These topics could be either extensions of, or new approaches to, material previously studied or content completely new to the students. Following is a list of suggested topics which would be included in this course:

ALGEBRA

1. Absolute value
2. Solving systems of equations using Cramer's Rule
3. Solving quadratic equations
4. Exponential and logarithmic functions

GEOMETRY

1. Finding areas
2. Distance formula in two and three dimensions
3. Pythagorean triples
4. Transformations in the plane

TRIGONOMETRY

1. Radian measure
2. Maclaurin expansion for sin x
3. Circular and trigonometric functions

PROBABILITY AND STATISTICS

1. Permutations
2. Combinations
3. Probability
4. Random numbers
5. Mean, median, and mode
6. Standard deviation
7. Binomial theorem

PRECALCULUS

1. Sequences and series
2. Complex numbers and polar coordinates
3. Vectors, lines, planes, and matrices
4. Mathematical induction
5. Approximations for roots and functions
   a. The location principle for solving equations
   b. The Bolzano bisection technique for locating roots
   c. The Maclaurin expansion for obtaining infinite series for functions

CALCULUS

1. Limits
2. Derivatives
3. Integrals

The prerequisite courses necessary for students to elect advanced computer mathematics would have to include Courses I, II, III and some type of precalculus course. Familiarity with a programming language is also essential.

For this publication the computer language will be BASIC, although other languages could be considered. BASIC was chosen because of its widespread availability on most computers. Another reason for choosing BASIC is the existence of a cadre of high school mathematics teachers who, already having received training in this language, would have the necessary knowledge to teach the course.

Advanced computer mathematics should be a full-year mathematics course. The classroom in which the course is taught should have sufficient microcomputers or terminals to enable students to work both independently or in small groups. In addition to formal classroom instruction, it would be desirable to have the students also assigned to a computer laboratory for one or two additional periods per week to work on class assignments or projects requiring programming. In this sense, the course structure would resemble a typical science class such as biology, chemistry, or physics in which the class includes a daily lecture/discussion session plus additional, scheduled laboratory time to perform experiments related to the daily classwork.

In any given week, consideration must be given to the time allotted to each of three activities:

mathematics content

programming techniques
using the computer in the laboratory

It is suggested that approximately 70% of the classroom instruction emphasize mathematics content and 30% programming techniques. An additional one or two periods per week should be provided for hands-on computer work. Activities should comprise exercises and problems related to

1. only the mathematics being studied;
2. only the programming techniques being studied; and
3. combinations of both mathematics and programming.

LESSONS AND PROBLEMS FROM ADVANCED COMPUTER MATHEMATICS

This section includes a series of lessons and some problems for computer solution designed to illustrate one of the topics listed in the course content outline. The particular sample presented details plans for four days of classroom instruction plus a laboratory assignment. For other topics, the amount of instructional time will vary depending on student familiarity with the topic and the prerequisite programming skills for developing a computer solution. For each problem, a five-part process is illustrated. This process consists of an analysis, a plan, a flowchart, a desired output, and the program (in BASIC code).

Topic: Complex Numbers and Polar Coordinates

Aims:

1. To convert points represented as ordered pairs from polar to rectangular coordinates and vice-versa.
2. To express in rectangular form a complex number given in polar form and vice-versa.

Performance Objectives: The student will be able to

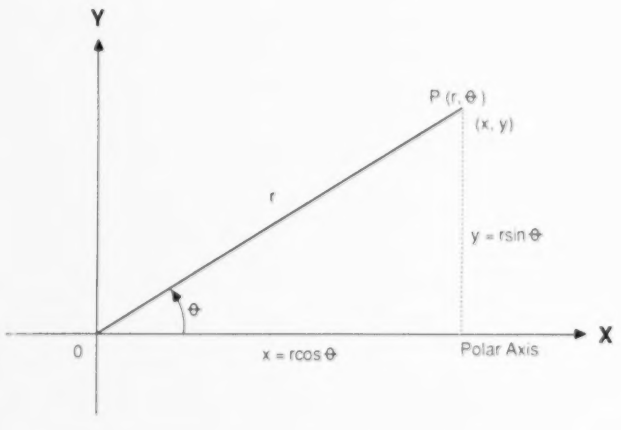
1. find the rectangular coordinates of a point named by given polar coordinates;
2. find the polar coordinates of a point named by given rectangular coordinates;
3. express a complex number in polar form;
4. express a complex number in rectangular form;
5. demonstrate his/her understanding of converting coordinate points from one set of coordinates to the other by developing a plan for the computer to perform the task;
6. demonstrate his/her understanding of expressing complex numbers in polar and rectangular form by developing a plan for the computer to perform the task.

Day 1

Motivation: Suppose we were to place a polar coordinate system on top of a rectangular coordinate system so that the origins coincided and the polar axis of the polar coordinate system coincided with the positive x-axis of the rectangular coordinate system. How would we be able to express a polar point such as (5,60°) in rectangular (x, y) form? How would we express a rectangular point such as (4, −8) in polar form?

Development:

a. Draw the two sets of axes, polar and rectangular, superimposed as indicated in the motivation statement and as shown below with an angle θ in standard position.



If P is any point in the plane, indicate that its polar coordinates are identified by the ordered pair (r,θ) as opposed to the usual notation with which the students are used to seeing for a point P in the rectangular coordinate system, namely (x, y). If we assume that the polar coordinates of point P are known, then the application of trigonometry can help us determine the rectangular coordinates of point P. If we assume that the coordinate system distance scales are equal in both systems, the rectangular coordinates (x, y) of the point P with coordinates (r,θ) would be found by determining

$$x = r\cos\theta \text{ and } y = r\sin\theta.$$

b. Illustrate this formulation with the example given in the motivation problem by converting polar point P (5,60°) into its rectangular coordinates (x, y); e.g.,

$x = r\cos\theta$	$y = r\sin\theta$
$x = 5\cos 60°$	$y = 5\sin 60°$
$x = 5\,(.5)$	$y = 5\,(.866)$
$x = 2.5$	$y = 4.33$

The rectangular coordinates of (5,60°) are (2.5, 4.33).

c. On the other hand, if we know the rectangular coordinates (x,y) of point P, then we can determine the polar coordinates of P by using the Pythagorean Theorem and the Arctangent function. Remind the students that the Arctangent function defines an angle in the first or fourth quadrants, so θ = Arctan (y/x) when the point P with rectangular coordinates (x,y) is in the first or fourth quadrant (where $x > 0$). Whenever (x,y) is in the second or third quadrants (where $x < 0$), it will be necessary to add 180° (π radians) to θ. It must also be remembered that whenever $x = 0$, θ will be $\pm$ 90° ($\pi/2$ radians). By applying the Pythagorean Theorem, we can determine that

$$r^2 = x^2 + y^2 \text{ and}$$

$$r = \sqrt{x^2 + y^2}$$

d. Illustrate this formulation with the example in the motivation problem by converting the rectangular point P (4, − 8) into its polar coordinates (r, θ).

$r = \sqrt{x^2 + y^2}$	θ = Arctan (y/x)
$r = \sqrt{4^2 + (-8)^2}$	θ = Arctan (−8/4)
$r = \sqrt{16 + 64}$	θ = Arctan (−2)
$r = \sqrt{80}$	$\theta = -63.4°$ or $\theta = 296.6°$
$r = 4\sqrt{5}$	

The polar coordinates for (4, −8) are ($4\sqrt{5}$, 296.6°).

e. Assign some application problems such as:

Find the polar coordinates of the point with the rectangular coordinates (−3,−3).

Find the rectangular coordinates of the point with the polar coordinates (−2,225°).

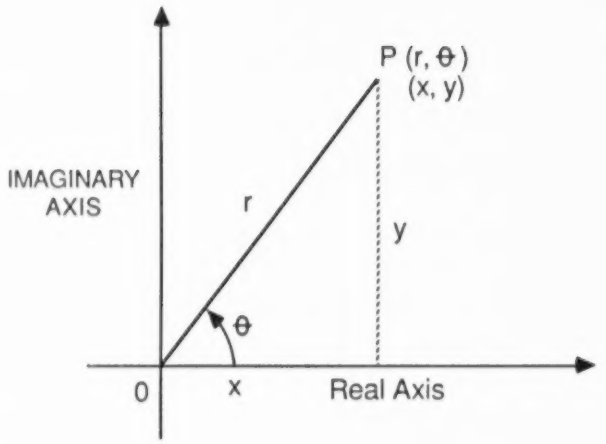
Day 2

Motivation: Suppose we are given the rectangular form of a complex number such as 5 + 12i. It can also be written as the ordered pair (5, 12). How can we write the rectangular form of the complex number 5 + 12i in polar form using the formulations r and θ developed previously?

Development:

a. Show how a complex number can be represented graphically as a point in the complex plane. A point P is graphed below corresponding to the complex number x + yi:

- horizontal axis = real axis
- vertical axis = imaginary axis
- angle θ is the amplitude
- r is the modulus



Once again, notice that from the previous figure above, $x = r \cos\theta$ and $y = r \sin\theta$. If we substitute the values for x and y in the rectangular form of the complex number x + yi, we will obtain the polar form of the complex number.

$$x + yi = r(\cos\theta + i \sin\theta).$$

b. Values for r and θ can be obtained as done previously; i.e.,

$r = \sqrt{x^2 + y^2}$ $\quad\quad$ θ = Arctan (y/x) when $x > 0$

θ = Arctan (y/x) + π when $x < 0$

It is customary to express θ in radian measure and place it in standard position.

c. Illustrate this formulation with the example given in the motivation problem by expressing 5 + 12i in polar form.

$r = \sqrt{x^2 + y^2}$	θ = Arctan (y/x)
$r = \sqrt{5^2 + 12^2}$	θ = Arctan (12/5)
$r = \sqrt{25 + 144}$	θ = Arctan (2.4)
$r = \sqrt{169}$	θ = 67.4° or 1.18 radians
$r = 13$	

The polar form of 5 + 12i is 13 (cos 1.18 + i sin 1.18).

d. Work another example of the alternate case; e.g., express 3(cos π + i sin π) in rectangular form x + yi.

$x = r \cos\theta$	$y = r \sin\theta$
$x = 3 \cos(\pi)$	$y = 3 \sin(\pi)$
$x = 3(-1)$	$y = 3(0)$
$x = -3$	$y = 0$

The rectangular form of the complex number 3(cos π + i sin π) is (−3,0).

e. Assign some application problems such as:

Express the complex number
12(cos (5 π /3) + i sin (5 π /3)) in rectangular form.

Express the complex number $2\sqrt{3} - 3i$ in polar form.

Day 3

Motivation: Present the following problem to the class for a computer solution. Given the polar coordinates of a point (r,θ), θ given in radians, have the computer find its rectangular coordinates (x, y).

Development:

a. Have students develop the five-step computer solution to the problem presented in the motivation.

Analysis: This could be completed as a whole-class discussion exercise. Some possible questions and answers that might constitute such a discussion are given below.

What information is given in the problem? (the polar coordinates of a point which are given as r and θ)

What values does the computer need to find? (the rectangular coordinates, x and y, of the point)

What process might be used to solve the problem? (the algorithm that can provide a solution to this problem is given by the equations

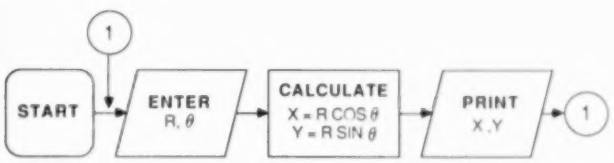
$x = r \sin \theta$ and $y = r \cos \theta$)

The BASIC programming statements SIN and COS and the procedure for rounding using INT might have to be reviewed or introduced by way of definition(s), proper format(s), and/or simple programming examples.

Plan: (This could be completed as a whole-class discussion exercise.)

1. Enter the values for r and θ (θ in radians).
2. Calculate: $x = r \cos \theta$ and $y = r \sin \theta$.
3. Print x and y as an ordered pair.
4. Go back to #1.

Flowchart: (This could be completed as a whole-class discussion exercise.)



Desired Output: (This could be completed as a whole-class discussion exercise.)

POLAR TO RECTANGULAR
(THETA IN RADIANS)

R	THETA	X	Y
2	1.05	1	1.73

Program: (This could be given to the students as a homework assignment.)

```
10  PRINT "POLAR TO RECTANGULAR"
20  PRINT "(THETA IN RADIANS)"
30  PRINT
40  PRINT "R"; TAB(6); "THETA"; TAB(12); "X"; TAB(18); "Y"
50  READ R, T : REM T (THETA IN RADS)
60  IF R = -99999 THEN 140
70  X=R*COS(T)
80  X=INT(X*100+.5)/100
90  Y=R*SIN(T)
100 Y=INT(Y*100+.5)/100
110 PRINT R; TAB(6); T; TAB(12); X; TAB(18); Y
120 GOTO 50
130 DATA ....., -.99999, -99999
140 END
```

It may be necessary to adjust for TAB, depending on the computer used.

b. The students should be asked to verify that their program solves the problem by using as data some of the problems given in previous homework assignments for which they would have the correct answers.

c. Extensions:

1. The students could also be asked to rewrite the program to be able to accept input of values of r and θ by the operator from the keyboard (change program to employ INPUT statement instead of READ and DATA statements) where θ could be entered as either radians or degrees.
2. The students could be asked to do the complete solution (analysis, plan, flowchart, planned output, and program) for the problem of converting rectangular coordinates to polar coordinates and the output could be written with either radian or degree measure for θ.

Day 4

Motivation: Present the following problem to the class: Given a complex number x + yi, have the computer express it in polar form.

a. Have students develop the five-step computer solution to the problem presented in the motivation.

Analysis: (This could be completed as a whole-class discussion exercise.)

Ask the students to review the formulas that are used to determine r and θ,

$r = \sqrt{x^2 + y^2}$

$\theta = \text{Arctan}\,(y/x)$ when $x > 0$

$\theta = \text{Arctan}\,(y/x) + \pi$ when $x < 0$

and the relationship between a complex number in rectangular and polar form, namely

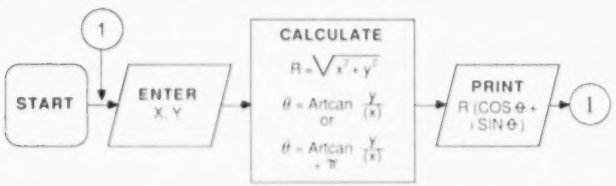
$$x + yi = r(\cos\theta + i \sin\theta)$$

The BASIC programming statements SQR and ATN might have to be reviewed or introduced by way of definition(s), proper format(s), and/or simple programming examples.

Plan: (This could be completed as a whole-class discussion exercise.)

1. Enter x and y.
2. Calculate r from $r = \sqrt{x^2 + y^2}$
3. Calculate θ from $\theta = \text{Arctan}(y/x)$ if $x > 0$ or $\theta = \text{Arctan}(y/x) + \pi$ if $x < 0$.
4. Print the polar form of the complex number using the $r(\cos\theta + i \sin\theta)$ – notation.
5. Go back to #1.

Flowchart: (This could be completed as a whole-class discussion exercise.)



Desired Output: (This could be completed as a whole-class discussion exercise.)

THE COMPLEX NUMBER IS −5 + 1I

ITS POLAR FORM IS 5.10(cos 3.34 + I sin 3.34)

Program: (This could be given to the students as a homework assignment.)

```
10  READ X,Y
20  IF X = -99999 THEN 160
30  R=SQR(X^2 +Y^2)
40  R=INT(R*100+.5)/100
50  IF X < 0 THEN 90
60  T=ATN(Y/X)
70  T=INT(T*100+.5)/100
80  GOTO 110
90  T=ATN(Y/X)+3.14159
100 T=INT(T*100+.5)/100
110 PRINT "THE COMPLEX NUMBER IS";X;" +
    ";Y;"I"
120 PRINT
130 PRINT "ITS POLAR FORM IS "; R; "(cos";T; +I
    sin";T)"
140 GOTO 10
150 DATA ....., -99999, -99999
160 END
```

b. The students should be asked to verify that their program solves the problem by using as data some of the problems given in previous homework assignments for which they would have the correct answers.

c. Extensions:

1. The students could also be asked to rewrite the program to be able to accept input of values of x and y by the user from the keyboard (change program to employ INPUT statement instead of READ and DATA statements) and the output could be written with either radian or degree measure for θ.
2. The students could be asked to do the complete solution (analysis, plan, flowchart, planned output, and program) for the problem of expressing a complex number given in polar form (θ should be able to be entered as either radians or degrees) into its rectangular form.

LABORATORY ASSIGNMENT

The following programming assignment could be given for the students to complete during their Advanced Computer Mathematics lab sessions. The due date would be a week or a week and a half from the date it is given to the students by the teacher.

Now that you have several programs for conversions among polar and rectangular coordinates and polar and rectangular forms of complex numbers, write a program that will allow the user to choose from among:

1. converting a rectangular point to a polar point
2. converting a polar point into a rectangular point
3. converting a complex number given in rectangular form into polar form
4. converting a complex number given in polar form into rectangular form.

The program must:

- make use of a menu for selecting the desired conversion and the GET command or the ON.....GOSUB command
- be "error-trapped"
- allow the user to input the appropriate value(s) of x, y, r, or θ (θ to be able to be entered as either radian or degree measure) and be sure the user is prompted as to when and how to input the values
- provide output which is self-explanatory
- allow the user, if he/she wants to run the program again for another set of data or end automatically if the user does not want to do so.

Against this background, the teacher would continue with a further development of complex numbers which would include simplifying of complex numbers (adding, subtracting, multiplying, and dividing complex numbers), products and quotients of complex numbers in polar form (e.g., developing the algorithm $r_1r_2(\cos(\theta_1+\theta_2) + i \sin(\theta_1+\theta_2))$, and powers and roots of complex numbers. The procedure would continue as outlined above, developing the necessary mathematical concepts and skills followed by problems for computer solution directly related to those topics and Advanced Computer Mathematics lab assignments designed to expand on either the mathematics concepts, the computer programming concepts, or both.

Additional Problems for Advanced Computer Mathematics

In the two examples that follow, the focus is on the five-part process leading to a computer solution to a problem related to the specific topic presented.

I. Topic: Polynomial Functions – The Integral Roots of the Polynomial Equation $P(x) = ax^3 + bx^2 + cx + d = 0$

Analysis:

The Factor Theorem provides a method for determining the roots of a polynomial equation. It states:

> A number r is a zero (root) of a polynomial function P(x) of degree n, $n > 0$, if and only if $x-r$ is a factor of P(x).

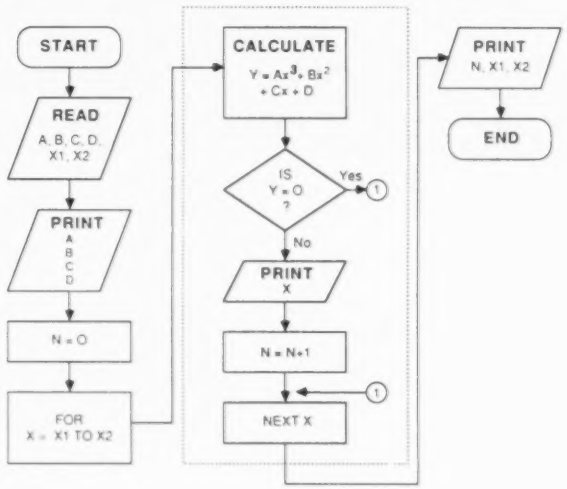
If the polynomial $P(x) = ax^3 + bx^2 + cx + d$ is evaluated for integral values of $x = r$ ($x - r = 0$), i.e., determining values for P(r), r an integer, then those values of r which make $P(r) = 0$ would be the roots of the polynomial P(x).

By making use of a loop, the computer could be made to "evaluate" the polynomial for each value of r in a specified range of consecutive, integral values of r, check to see if the value of $P(r) = 0$, and print out those values within the specified range of integers chosen which would be the zeros of the polynomial function. A discussion should be included to select the range of values of r and the incrementation.

Plan:

1. Enter values for A, B, C, D, the coefficients of the polynomial P(x), and values for X1 and X2, the lower and upper limits of the range of integers to be used to evaluate P(x) successively.
2. Calculate a value of the polynomial for $x_1 \le r \le x_2$, r an integer.
3. Check to see if $P(r) = 0$.
4. If $P(r) = 0$, print out the result that r is a root of P(x).
5. Go back to #2 for the next integral value of r.

Flowchart:



Desired Output:

ROOTS OF THE POLYNOMIAL EQUATION AX^3 + BX^2 + CX + D = 0
A = 2
B = −3
C = 2
D = −8
2 IS A ROOT

THE EQUATION HAS 1 INTEGRAL ROOT BETWEEN −5 AND 5

Program:

```
10  PRINT "ROOTS OF THE POLYNOMIAL"
15  PRINT "EQUATION AX^3 + BX^2 + CX + D = 0"
20  READ A,B,C,D,X1,X2
30  IF A = - 99999 THEN 190
40  PRINT "A = ";A
50  PRINT "B = ";B
60  PRINT "C = ";C
70  PRINT "D = ";D
80  N = 0
90  FOR X = X1 TO X2
100 Y=A*X*X*X+B*X*X+C*X+D
110 IF Y< >0 THEN 150
120 PRINT
130 PRINT X; "IS A ROOT"
140 N=N+1
```

```
150 NEXT X
160 DATA....., -9999, -9999, -9999, -9999, -9999,
    -9999
170 PRINT
180 PRINT "THE EQUATION HAS ";N;" ROOT(S) BE-
    TWEEN ";X1;" AND ";X2
190 END
```

II. Topic: Sequences and Series – Finding the Arithmetic Means Between Any Two Real Numbers

Analysis:

By definition, the term(s) between any two nonconsecutive terms of an arithmetic sequence are called the arithmetic means. If the first of the nonconsecutive terms is represented by a, the second of the nonconsecutive terms by a_n, and the total number of terms in the sequence is the number of arithmetic means plus two, then the general formula for the n_{th} term of an arithmetic sequence can be used to solve for d, the constant, common difference between consecutive terms of the arithmetic sequence. The n_{th} term of an arithmetic sequence with first term a and common difference d is given by the formula:

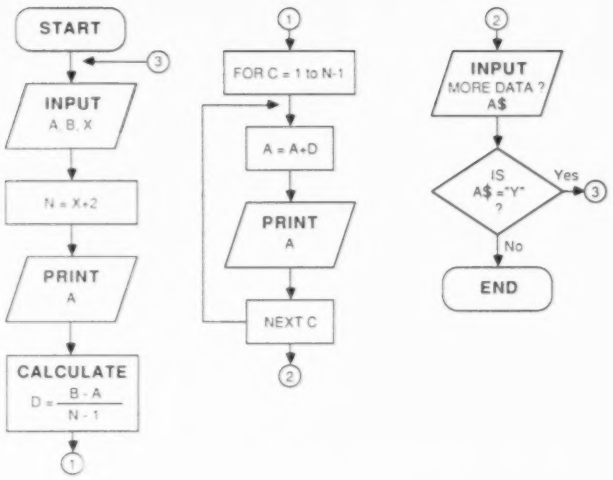
$$a_n = a + (n-1)d$$

Therefore, $d = (a_n - a)/(n-1)$.

The computer can calculate d. Beginning with the first term of the sequence, the value of d can be added by means of a loop to each successive term until it reaches the last term. This process generates the arithmetic means between the two given nonconsecutive terms of the sequence.

Plan:

1. Enter values for A and B, the two nonconsecutive terms of the arithmetic sequence and enter a value for X, the number of arithmetic means desired between A and B.
2. Calculate a value for D, the constant difference between the successive terms of the arithmetic sequence.
3. Add the value of D to A and print the result.
4. Add the value of D to the value obtained in #3, print it, and continue in this manner for a total of x + 2 terms where the x+2nd term would be B.



Desired Output:

```
ARITHMETIC MEANS
ENTER THE TWO NONCONSECUTIVE TERMS
OF THE ARITHMETIC SEQUENCE - - - A,B?1,4
HOW MANY ARITHMETIC MEANS DO YOU
WANT BETWEEN 1 AND 4 (MUST BE A
NUMBER > =1)?2
1    2    3    4
THE SEQUENCE ABOVE LISTS YOUR TWO NON-
CONSECUTIVE NUMBERS AND THE
2 ARITHMETIC
MEANS BETWEEN THEM.
DO YOU WANT TO COMPUTE SOME MORE
ARITHMETIC MEANS (Y OR N)?
?N
```

Program:

```
10  HOME
20  PRINT "ARITHMETIC MEANS"
30  PRINT:PRINT
40  PRINT "ENTER THE TWO NONCONSECUTIVE
    TERMS OF THE":PRINT"ARITHMETIC SE-
    QUENCE - - A,B";
50  INPUT A,B
60  PRINT
70  PRINT "HOW MANY ARITHMETIC MEANS DO
    YOU WANT BETWEEN "; A; "AND "; B; " (MUST
    BE A NUMBER >=1)";
80  INPUT X
90  N=X+2
100 PRINT:PRINT
110 PRINT A; " ";
120 D=(B-A)/(N-1)
130 FOR C=1 TO (N-1)
```

```
140 A=A+D
150 PRINT A;" ";
160 NEXT C
170 PRINT:PRINT:PRINT
180 PRINT "THE SEQUENCE ABOVE LISTS YOUR
    TWO NONCONSECUTIVE": PRINT "NUMBERS
    AND THE ";X;" ARITHMETIC MEANS BE-
    TWEEN THEM."
190 PRINT:PRINT
200 PRINT "DO YOU WANT TO COMPUTE SOME
    MORE ARITHMETIC MEANS (Y OR N)?"
210 INPUT A$
220 IF A$="Y" THEN 10
230 END
```

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SECTION V

Integrating Commercially Produced Software into the Mathematics Curriculum

The availability and potential uses of software are important aspects in the process of deciding how to use the microcomputer as an instructional tool in the mathematics classroom. As an aid in this process, two tables of software data are presented in this section.

SOFTWARE LISTINGS

All items listed in these tables have been reviewed by mathematics teachers and supervisors. However, these lists are not inclusive. They contain software submitted to the writers of this document which were considered appropriate for the listed course content areas. The first is an alphabetical listing of many examples of current commercial software. The second references the software in Table 1 to subjects from the high school mathematics curriculum. There are specific suggestions following the two tables for infusing commercially produced software into the following courses: *General High School Mathematics*, Course I, Course II, and Course III of the *Three-Year Sequence for High School Mathematics*.

The numbered alphabetical listing of software in Table 1 has been prepared to help you easily identify the software publisher, the type of instructional content, and the microcomputers for which the software is designed.

Key For Table 1

Type:	**D & P**	**Drill and Practice**
	T	**Tutorial**
	P	**Problem Solving**
	D	**Demonstration**
	I	**Instructional Game**
	S	**Simulation**
	M	**Management**
Machine:	**A**	**Apple**
	AT	**Atari**
	C	**Commodore**
	I	**IBM**
	T	**Tandy**

As indicated, some of the software listed in Table 1 is available for different microcomputers. Occasionally there are slight variations in the quality of a piece of software developed for different microcomputers. Therefore, it is recommended that you preview the software that has been specifically designed for your computer system.

Subjects and specific content topics from the high school mathematics curriculum are outlined in Table 2. The software is referenced to the topics by the number used in the alphabetical listing of Table 1.

These reference lists are presented as an aid to secondary mathematics educators in locating software for possible use in the curricula. We recommend that all programs be previewed and evaluated (see Part I, Evaluation and Acquisition Criteria) to determine their suitability for specific instructional purposes.

TABLE 1

SOFTWARE

	TITLE	PUBLISHER	TYPE	MACHINE	COPYRIGHT
1.	Adding Decimals	Microcomputer Workshops Courseware	D&P	A,AT,I,T,C	
2.	Adding Fractions	Microcomputer Workshops Courseware	D&P	A,AT,I,T,C	
3.	Advanced Placement Calculus	Intellectual Software	T	A	1985
4.	Algebra 1	Edu-ware	D&P,T	A,C,I	1982, 1985
5.	Algebra 2	Edu-ware	D&P,T	A,C,I	
6.	Algebra 3	Edu-ware	D&P,T	A,C,I	
7.	Algebra 4	Edu-ware	D&P,T	A,C,I	1982, 1985
8.	Algebra 5 & 6	Edu-ware	D&P,T	A,C,I	
9.	Algebra Billiards	Curriculum Applications	I	A	1984
10.	Algebra Coach	Educational Activities, Inc.	T,D&P,P,D	A	1985
11.	Algebra Drill & Practice I	CONDUIT	D&P	A	1980, 1981
12.	Algebra Drill & Practice II	CONDUIT	D&P	A	1982
13.	Algebra Pak	Sliwa Enterprises, Inc.	T,D&P	I	1984
14.	Algebraic Expressions	Educational Activities, Inc.	T,D&P,P,D	A,T,P,C	1982
15.	Algebraic Word Problems I: Motion	Microcomputer Workshops Courseware	T,D&P,P,D	A	1985
16.	Algebraic Word Problems II: Area, Perimeter, & Lever	Microcomputer Workshops Courseware	T,D&P,P,D	A	1985
17.	Algebraic Word Problems III: Mixture, Coin, & Investment	Microcomputer Workshops Courseware	T,D&P,P,D	A	1985
18.	Algebraic Word Problems IV: Percent, Mixture, & Age	Microcomputer Workshops Courseware	T,D&P,P,D	A	1985
19.	Algernon	Sunburst Communications, Inc.	I	C	1985
20.	Area & Perimeter	Britannica Educ. Corp.	D&P	A	1986
21.	Arith-Magic	Quality Educational Design (QED)	I	A,C,T	1983
22.	Arith-Magic II	Quality Educational Design (QED)	I	A,C,T	1984
23.	Arithmetic Doctor	Educational Activities, Inc.	I	A,AT,C,T	1986
24.	Ato Grade: A	Dennison	I	A	
25.	Balancing Acts	Intellectual Software	I	A	
26.	Basic Math Competency Skill Building	Educational Activities, Inc.	D&P,T	A,AT,C,T	1986
27.	Basic Mathematics Skills	Media Materials, Inc.	D&P	A	1985
28.	Beginning Algebra	Programming Design, Inc.	D&P,T	A,AT,C	1984
29.	Binomial Multiplication	Microcomputer Workshops Courseware	T,D&P	A,I,T,C	1984
30.	Calculus Tool Kit	Addison-Wesley Publishing Company, Inc.	D,T	I	1986
31.	Challenge Math	Sunburst Communications, Inc.	I	A	1983
31a.	Computerized Regents Assistance Method-Course I	CRAM Software	D&P,T	A	1987
32.	Computers and Math	Scholastic Software	P,D	A	1986

	TITLE	PUBLISHER	TYPE	MACHINE	COPYRIGHT
33.	Decimals: Add and Subtract	Gamco Industries, Inc.	D&P,T	A	1984
34.	Decimals: Multiplication and Division	Gamco Industries, Inc.	D&P,T	A	1984
35.	Decimals 2: Multiplication of Decimals	Scott Foresman and Company	D&P,T	A	1985
36.	Decimals 3: Division of Decimals	Scott Foresman and Company	D&P,T	A	1985
37.	Decimal Skills	Media Materials, Inc.	D&P,T	A	1985
38.	Discovery Learning in Trigonometry	CONDUIT	S,I,D	A	1985
39.	Discrete Probability	COMPress	T,S,I,D	A	1983
40.	Dividing Decimals	Microcomputer Workshops Courseware	D&P,T	A,AT,I,T,C	
41.	Dividing Fractions	Microcomputer Workshops Courseware	D&P,T	A,AT,I,T,C	
42.	Division Skills	Media Materials, Inc.	D&P,T	A	1985
43.	Elementary Reasoning Skills	Intellectual Software	P	A	
44.	E.M.E.Scientific Notation	Educational Materials & Equipment Company	T,D	A	1985
45.	Equations	Microcomputer Workshops Courseware	D&P,T	A,AT,I,T,C	1984
46.	Equations II	Microcomputer Workshops Courseware	T,D&P	A,I,T,C	1984
47.	Equations III	Microcomputer Workshops Courseware	T,D&P	A	1985
48.	Estimating & Common Sense	Imperial International Learning Corporation	P	A	1984
49.	Exploring Triangles	Britannica	T	A	1985
50.	Factoring Algebraic Expressions	Microcomputer Workshops Courseware	T,D&P	A,I,T,C	1984
51.	Fractional Series	Quality Educational Design (QED)	D&P,T	A,C,T	1983
52.	Fractions 4: Mixed Numbers	Scott Foresman and Company	D&P,T	A	1985
53.	Fractions 5: Multiplication & Division of Fractions	Scott Foresman and Company	D&P,T	A	1985
54.	Fractions Addition & Subtraction	Gamco Industries, Inc.	D&P,T	A	1985
55.	Fractions Decimals and Percents	Intellectual Software	D&P,T	A	1985
56.	Fundamentals of Math	Sliwa Enterprises, Inc.	D&P	A,I	1985
57.	Fundamentals of Probability and Statistics	Educational Activities, Inc.	T	A	1986
58.	Geometry	Sliwa Enterprises, Inc.	D&P,T	A,I	1986
59.	Geometry Alive!	Educational Activities, Inc.	D&P,T	A,AT,C,T	1984
60.	Geometry Angles of Triangles & Polygons	Scott Foresman and Company	T	A	1985
61.	Geometry: Areas of Circles	Intellectual Software	T	A	1985
62.	Geometry: Congruent Triangles	Scott Foresman and Company	T	A	1985
63.	Geometry: More Congruent Triangles	Scott Foresman and Company	T	A	1985
64.	Geometry: Quadrilateral	Scott Foresman and Company	T	A	1985
65.	Geometry With Parallel Lines	Intellectual Software	T	A	1985
66.	Get To The Point	Sunburst Communications, Inc.	P	A	1984
67.	Graph Challenge	COMPress	I	A	1985
68.	Graphing Equations	CONDUIT	T,D,D&P	A	1985
69.	Graphing Linear Functions	Microcomputer Workshops Courseware	T,D&P,D	A,I,T,C	1984
70.	Green Globs & Graphing Equations	Sunburst Communications, Inc.	I,P,D,M	A	1986

	TITLE	PUBLISHER	TYPE	MACHINE	COPYRIGHT
71.	H.S.Math Competency Series	Microcomputer Workshops Courseware	D&P	A,AT,I,T,C	
72.	H.S.Math Competency Series (Sp.)	Microcomputer Workshops Courseware	D&P	A,AT,I,T,C	
73.	Integers: Fast Facts	EduSoft	I	A	1983
74.	Intermediate Algebra	Avant-Garde Creations	T,D	A	1984
75.	Intermediate Reasoning Skills	Intellectual Software	P	A	1985
76.	Interpreting Graphs	CONDUIT	T	A	1982
77.	Interpreting Graphs	Sunburst Communications, Inc.	S,I	A	1986
78.	Intro. Software for Algebra 1	Addison-Wesley Publishing Company, Inc.	D&P,T	A	1985
79.	Introduction to Trigonometry	Queue	T,D&P	A	1983
80.	Introductory Algebra	Avant-Garde Creations	T,D&P	A	1984
81.	Introductory Calculus	Microphys Programs, Inc.	T	I	1984
82.	Language of Math	Krell Software	T	A	1984
83.	Learning Addition	Classroom Consortia Media, Inc.	D&P	I	1983
84.	Learning Division	Classroom Consortia Media, Inc.	D&P	I	1983
85.	Learning Multiplication	Classroom Consortia Media, Inc.	D&P	I	1983
86.	Learning Place Value	Classroom Consortia Media, Inc.	D&P	I	1983
87.	Learning Subtraction	Classroom Consortia Media, Inc.	D&P	I	1983
88.	Lessons In Algebra	George Earl	T	A	1981
89.	Linear Equations	Krell Software	T	A	1983
90.	Linear Functions and Graph II	Queue	T,D&P,P	A	1984
91.	Logic Workout	Avant-Garde Creations	I	A	1984
92.	Managing Lifestyles	Sunburst Communications, Inc.	T	A,T	1984
93.	M*A*T*H The Fundamentals: Operations	Krell Software	D&P	A	1983
94.	Math Diagnostic	Krell Software	M	A	1985
95.	Math Football	Gamco Industries, Inc.	I	A	1985
96.	Math Skills Junior High Level	Britannica Educ. Corp.	D&P	A	1983
97.	Mathematics Today: Management System	Harcourt Brace Jovanovich	M	A	1985
98.	Mathematics Today: Problem Solving	Harcourt Brace Jovanovich	P	A	1985
99.	Metric I Fundamentals of Decimals	Classroom Consortia Media, Inc.	D&P	I	1983
100.	Metric II Applications	Classroom Consortia Media, Inc.	D&P	I	1983
101.	Metric III Units of Measure	Classroom Consortia Media, Inc.	D&P	I	1983
102.	Metric IV Linear Units	Classroom Consortia Media, Inc.	D&P	I	1983
103.	Metric V Area/Density	Classroom Consortia Media, Inc.	D&P	I	1983
104.	Microcomputer Software: Percents	Media Materials	D&P	A	1985
105.	Mixed Numbers	Media Materials	D&P	A	1985
106.	Multiplication and Division	Gamco Industries, Inc.	D&P	A	1984
107.	Multiplication Fractions	Microcomputer Workshops Courseware	D&P	A,AT,I,T,C	1984
108.	Multiplying Decimals	Microcomputer Workshops Courseware	D&P	A,AT,I,T,C	1984

	TITLE	PUBLISHER	TYPE	MACHINE	COPYRIGHT
[illegible]	Order of Operations	Microcomputer Workshops Courseware	D&P	A,AT,I,T,C	1984
110.	Percent 1	Scott Foresman and Company	D&P	A	1985
111.	Percent 2:Using Percents	Scott Foresman and Company	D&P	A	1985
112.	Polynomial Practice	Sunburst Communications, Inc.	D&P,I	A	1983
113.	Practical Geometry	Intellectual Software	T	A	1985
113a.	Plane Geometry	Educational Courseware	T	A	1985
114.	Problem Solving in Algebra	Britannica Educ. Corp.	D&P,P	A,T	1983
115.	Probability 1	Microphys Programs, Inc.	D&P,M	A,I,T,C	1985
116.	Proportions & Percents	Quality Educational Design	D&P	A,C,T	1981
117.	Proportions & Per Cents	Quality Educational Design	D&P	A,C,T	1980-1981
118.	Quadratic Equations II	Queue	T,D&P	A	1985
119.	Ratio and Proportion	Microcomputer Workshops Courseware	T,D&P,P	A	1985
120.	Ratios and Proportions	Media Materials	D&P	A	1985
121.	Read And Solve Math Prob. 2	Educational Activities, Inc.	D&P,T	A,AT,C,T	1983
122.	Rounding	Gamco Industries, Inc.	D&P	A	1984
123.	SAT Skill Series: Math Skills Pak	Sliwa Enterprises, Inc.	D&P	A	1985
124.	SEI Math Series:Geometry	Sliwa Enterprises, Inc.	D&P	A	1985
125.	Sem Calc	Sunburst Communications, Inc.	D&P	A	1983
126.	Senior High Math I M1	Microphys Programs, Inc.	D&P	I	1984
127.	Senior High Math II M2	Microphys Programs, Inc.	D&P	I	1984
128.	Simple Inequality	Intellectual Software	D&P,T	A	1985
129.	Simultaneous Linear Equations	Microcomputer Workshops Courseware	D&P,T	A,I,T,C	1984
130.	Solving Equations	Educational Activities, Inc.	D&P,T	A,AT,T,C	1981
131.	Solving Equations & Inequalities	Sunburst Communications, Inc.	T,D&P,S	A	1983
132.	Solving Fractional Equations	Microcomputer Workshops Courseware	T,D&P	A	1985
133.	Solving Quadratic Equations	Microcomputer Workshops Courseware	T,D&P	A,I,T,P,C,AT	1985
134.	Strategies in Problem Solving: Spinners & Slugs	Scott Foresman and Company	P,I	A	1986
135.	Strategies in Problem Solving: Dinosaurs & Squids	Scott Foresman and Company	P,I	A	1985
136.	Subtracting Decimals	Microcomputer Workshops Courseware	D&P	A,AT,I,T,C	1985
137.	Subtracting Mixed Fractions	Microcomputer Workshops Courseware	D&P	A,AT,I,T,C	1985
138.	Super Math	Educational Software, D.C. Heath & Company	D&P	A	1983
139.	Superplot	EduSoft	D	A	1985
140.	Surfaces for Multivariable Calculus	CONDUIT	D	A	1982
141.	Targets	Sunburst Communications, Inc.	I	A,AT,T	1985
142.	Teasers By Tobbs	Sunburst Communications, Inc.	I	A,AT,C,I,T	1982
143.	The Geometric Supposer: Quadrilaterals	Sunburst Communications, Inc.	S	A	1985

	TITLE	PUBLISHER	TYPE	MACHINE	COPYRIGHT
144.	The Geometric Supposer: Triangles	Sunburst Communications, Inc.	S	A	1985
145.	The Language of Math: Rates & Ratios	Krell Software	T	A	1984
146.	The Super Factory	Sunburst Communications, Inc.	T	A	1985
147.	The Whatsit Corporation	Sunburst Communications, Inc.	I	A,T	1983
148.	Tobbs Learns Algebra	Sunburst Communications, Inc.	I,P	A	1983
149.	Trigonometry	Sliwa Enterprises, Inc.	D&P	I	1985
150.	Trigonometry of the Right Triangle	Microcomputer Workshops Courseware	T,D&P,P	A	1985
151.	WEPCO Electricity:Study Guide for Precalculus Algebra	COMPress	T,D&P,D	A	1983
152.	WEPCO Electricity: Study Guide for Trigonometry	COMPress	T,D&P,S,P,D	A	1984
153.	Whole Numbers: Add and Subtract	Gamco Industries, Inc.	D&P	I	1985

TABLE 2

COURSE/SOFTWARE MATRIX

COURSE	SOFTWARE
GENERAL HIGH SCHOOL MATHEMATICS	
I. Set of Integers	
II. Rational Numbers	1,2,21,22,23,27,31,33,34,35,36, 37,40,41,42,52,53,54,55,71,72,83, 84,85,86,87,93,94,96,97,99,105, 106,107,108,123,136,137,138,153
III. Graphing	26,67,139
IV. Measurement of Geometric Figures	20,25,60,61,100,101,102,103
V. Ratio, Proportion, and Percent	26,104,110,111,119,120,145
VI. Probability and Statistics	26,39,57,115,134
VII. Consumer and Job Related Mathematics	44,92,146,147
VIII. Informal Geometry	49,59,64,65,66,113,124,139,143
IX. Recreational Mathematics	95,135,141,142
X. Flow Charts and Calculators	32
XI. Volume of Geometric Solids	
XII. Mathematical Reasoning	48,75,82,91,98,121
SEQUENTIAL MATHEMATICS, COURSE I	
• Logic	75,123,126
• Algebra	4,5,6,7,8,9,10,11,12,13,14,15,16, 17,18,19,20,28,29,46,47,50,73,78, 80,88,89,109,112,114,116,117,118, 128,130,131,132,133,145,148
• Plane Euclidean Geometry	49,58,59,60,61,62,63,64,65,143,144
• Transformation Geometry	
• Analytic Geometry	67,68,70,76,77,139
• Probability	39,57,115
• Statistics	57,76,77
SEQUENTIAL MATHEMATICS, COURSE II	
• Logic	75
• The Algebra of Systems	
• Euclidean Geometry	49,59,60,62,63,64,65,66,124,143,144
• Analytic Geometry	67,68,69,70,90,139
• Solution of Quadratic Equations	118
• Probability and Permutations, Combinations	39,57
SEQUENTIAL MATHEMATICS, COURSE III	
• Complex Numbers	
• Relations and Functions	51,69,90
• Geometry of the Circle	
• Trigonometry	39,79,149,150,152
• Probability	39,57
• Statistics	57

SUGGESTED SOFTWARE USES

Four pieces of software have been selected for a more detailed description of their content, including suggested uses within the curriculum. They are as follows: (1) *Basic Math Competency Skill Building* for General Mathematics, (2) *Strategies In Problem Solving – Spinners and Slugs* for Course I of Sequential Mathematics, (3) *The Geometric Supposer: Quadrilaterals* for Course II of Sequential Mathematics, and (4) *Superplot* for Course III of Sequential Mathematics.

1. General Mathematics:*Basic Math Competency Skill Building* (Educational Activities, Inc.), Table 1, #26.

   The *Basic Math Competency Skill Building* series is a group of tutorial/drill and practice programs designed to review and reinforce many basic skills found in the *General High School Mathematics* course of study.

   Since each lesson is identified on the menu and in the management section, it is easy to follow the specific skills on which the student is working. Correct responses bring immediate positive reinforcement; incorrect responses lead to specific instructional branches based on the error made. The student controls the pace throughout the program and can study any message for as long as desired.

   At the end of each lesson, the student takes a quiz. A score of 70% or above receives an appropriate congratulatory message and/or graphic reward. If a student scores below 70%, she/he is advised to do some additional practice examples or to review the tutorial. The management system records each student's most recent scores for each lesson. Since all the numbers are randomly generated, the lessons may be repeated as often as necessary.

   This software is most effectively used by students working individually in a computer lab. The teacher should monitor use by providing the students with individual contracts or prescriptions that meet their specific needs. After a session in the computer laboratory, the teacher may view the students' scores on the screen or on a printout. These printouts can be used by the teacher to monitor the progress of each student and plan subsequent assignments.

2. Sequential Mathematics – Course I: *Strategies in Problem Solving – Spinners and Slugs* (Scott Foresman), Table 1, #134.

   One of the most important objectives in the learning of mathematics is developing the ability to solve problems. We are well aware that problem solving needs to be an integral part of the mathematics curriculum. *Strategies in Problem Solving – Spinners and Slugs* presents an opportunity for the instruction and practice that students need to become better problem solvers. This type of software may be used successfully with students from grades 5 through 9 since it contains probability problem-solving activities that will challenge students from the beginning to the advanced level.

   The *Strategies in Problem Solving* program can be used in three different ways: (1) as a classroom teaching aid, (2) in small group discussions, and (3) for individual use. Each of the problems on the disk can be completed within 15 to 30 minutes depending on the skill levels of the students. The teacher's guide that accompanies the software contains detailed and appropriate suggestions for each of these three settings. It also contains references to lead-in activities and follow-up activities, along with 10 worksheets and their answers.

   In a classroom demonstration it is possible to go through the main problems more than once, using different data. This will allow the teacher to emphasize the importance of the process of solution and the strategies involved. Small group discussions and work with the software may follow a classroom presentation or be used in place of the classroom presentation. This approach is a good one for enabling all students to take a more active part in sharing and verbalizing their strategies. It would also be appropriate for any size computer lab. On an individual basis, when computer time is available outside of class time, students may use the software for extra practice and enrichment.

   Students should have an understanding of the content of the probability of an event's occurring. If they have not used this program before, they should start with the "Great Turtle Race" problem in which a special pair of dice is used. It contains a strategy menu with nine options: (1) "give you answer," (2) "read the problem again," (3) "get a definition," (4) "get a hint," (5) "make a table," (6) "see a table," (7) "use the calculator," (8) "write a BASIC program," and (9) "see a solution." Once this problem is completed or the solution is viewed, the students are advanced to the review problem, "Slithering Slugs," where two spinners are used. It is also possible to "Answer the Computer's Challenge" or "Challenge the Computer" in the section of the program called "Create a Problem."

   This is an excellent program for enhancing a student's problem-solving strategies and improving their understanding of probability. It could be used when beginning a unit on probability in Course I of the *Sequential Mathematics* curriculum.

3. Sequential Mathematics - Course II: *The Geometric Supposer: Quadrilaterals* (Sunburst), Table 1, #143.

   The Geometric Supposer: Quadrilaterals allows students to make any construction they wish on any quadrilateral. The program records that construction as a procedure which can then be executed with any other quadrilateral. As a result, the user can explore whether the consequences of a given construction on a given quadrilateral are dependent on some particular property of that quadrilateral, or if the result can be generalized.

   While proof still remains critical to both the creating and learning of mathematics, *The Geometric Supposer* assists in allowing conjecture and exploration to assume their proper role as key activities in the teaching and learning of geometry in Course II of *Sequential Mathematics*.

   The Geometric Supposer is a useful tool for introducing new geometric concepts, evoking ideas, answering questions, and exploring problems with the whole class. When there is only one computer in the classroom, it is useful to have a large screen monitor in the front of the room or several small monitors around the room, so that everyone in the class can see the display.

   A teacher can select problems to provide the class with a challenge and can manage the discussion so that the students see themselves as part of a team trying to solve a puzzle. This kind of discussion can provide students with a model for devising and testing hypotheses and can lay the foundation for theorems and proofs.

   If students can work on the computer on their own in a laboratory setting, a key role for the teacher during the class time is to help the students analyze their data and findings. The teacher becomes a partner in the conversation, helping students focus their arguments and look for evidence in their data. The process of having students report to their classmates can be, and often is, an occasion for sharply differing opinions about some geometric relationships.

   Teachers can use lab time effectively in two different ways:

   - by asking the students to work on specific problems in the lab on their own, outside of class time, for two or three periods per week; or
   - by holding a class in the lab one or two times per week and using the remaining class time to introduce new topics and discuss findings from the lab.

   The level of structure and guidance that the teacher provides will depend on an assessment of the students. Written problem sets and assignments are essential to guide student work.

   After students have worked on a problem on their own, the teacher can start the follow-up discussion by taking a student's findings and putting them on the chalkboard or an overhead transparency. The teacher can ask the class if they found similar data and can ask what conclusions they might make from that data. The teacher should ask the students to be explicit about their approaches, how they thought about the problem, and why they chose to make a particular construction or measurement on a specific quadrilateral. The teacher might want to see if the students took different approaches. Students may be challenged with examples that they have not considered and asked if they see patterns emerging from the data.

   At first the students' arguments are likely to be informal. However, as students become more comfortable and familiar with the subject matter and the computer program, the teacher should expect written formal proof.

   Lab work with *The Supposer* is most successful when students work in pairs. With a sufficient number of computers, students can work individually, but because of the open-ended nature of the inquiry, the educational experience is likely to be richer when students can put their heads together and are forced to subject their ideas and actions to another's scrutiny. In cases in which there are too few computers for the students to work in pairs, finding a way to schedule smaller groups of students is advisable.

4. Sequential Mathematics – Course III: *Superplot* (EduSoft), Table 1, #139.

   Superplot graphs any polynomial, trigonometric, logarithmic, or exponential function – one, two, or even five at a time. These functions may be superimposed on the same set of coordinate axes for comparison. The program has optional sound and color. The graphs may be plotted in a different color to make the comparisons easier.

   Once functions have been graphed, they may be scrolled left, right, up, or down. It is also possible to zoom in or out on either axis or both, simultaneously. Limits, functions, and graphing modes can be easily changed. With *Superplot*, input and output of functions is in standard algebraic notation.

   Superplot would be very beneficial to use for a classroom demonstration in the circular functions unit of *Sequential Mathematics* Course III. Specifically, this program would be a great aid to both the

teacher and the student for the graphing of trigonometric equations while studying the effect of a (amplitude) and b (period) in the equations $y = a \sin bx$ and $y = a \cos bx$. To plot these graphs accurately at the board or on an overhead transparency takes quite a bit of time. *Superplot* is an excellent alternative.

Using *Superplot*, you could graph $y = 3 \sin x$, $y = 1/3 \sin x$, and $y = 6 \sin x$ very quickly on the same set of axes and clearly demonstrate the effect of a change in the amplitude.

Likewise, simultaneous graphing of

$y = 3 \cos x$,
$y = 3 \cos (3x)$, and
$y = 3 \cos (1/3\ x)$

would give students a very clear, very accurate, and very rapid notion of the effect of the change in the period of a trigonometric function. The class could discuss the limits which might need to be set on the values of the x-axis and y-axis as the program is being set up, prior to the actual entry and plotting of the equation.

Superplot would also be useful when reviewing various types of trigonometric graphing problems typically found on Part II of the Course III Regents Examination.

Example: a) On the same set of axes, sketch and label the graphs of the equation $y = 2 \sin (1/2\ x)$ and $y = \cos 2x$ in the interval $0 \leq x \leq 2\pi$.

b) What is the value of x in the interval $0 \leq x \leq 2\pi$ for which $2 \sin (1/2\ x) - \cos 2x = 1$?

Using the zoom feature, an in-depth analysis of part (b) of the above question can be done. A whole series of problems of this type can be done more quickly and efficiently using a program such as *Superplot*. It provides the students with a very comprehensive look at the types of results to expect.

CONCLUDING REMARKS

Many technologies have found their way into the schools. Tape recorders brought about language laboratories, televisions brought instructional programming, and calculators added a new dimension to mathematics instruction. All of these innovations have had varying degrees of success in the classroom. Now we have computers. There is no question that computers will continue to have a great impact on our lives.

This guide has presented various ways that computers can be used to increase learning in mathematics. The material presented in this document represents good teaching and good mathematics. Naturally, using computers is not the only way to enhance mathematics instruction. But, this technology offers interesting possibilities that are only bounded by the creativity of teachers. Through their creativity, teachers will, in using computers, open new avenues for learning to students.

This document is designed to provide teachers with a resource for using computers in the classroom so that computers become an effective tool for mathematics instruction.

APPENDIX A

Land Management Program

PROGRAM

```
5     REM ****STATE****
10    PRINT "THIS IS A SIMPLIFIED PROGRAM TO
      SHOW YOU HOW"
20    PRINT "COMPUTERS CAN GIVE PRACTICAL
      ANSWERS TO"
30    PRINT "MATHEMATICAL MODELS IN EVERY-
      DAY LIFE"
40    PRINT
50    PRINT "HERE WE ARE ASKED INFORMA-
      TION ABOUT HOW MANY"
60    PRINT "ANIMALS OF A GIVEN TYPE, CAN BE
      SUSTAINED BY A"
70    PRINT "GIVEN PARCEL OF LAND. THIS IS DE-
      TERMINED BY"
80    PRINT "FIRST ENTERING THE MINIMUM
      NUMBER OF UNITS EACH"
90    PRINT "ANIMAL MUST HAVE TO SURVIVE."
100   PRINT "PLEASE PRESS ANY KEY TO CON-
      TINUE"
110   LET A $=INKEY$
120   IF A $=***THEN GOTO 110***
130   REM "INPUT DATA"
140   CLS
150   INPUT "WHAT ANIMAL WILL YOU BE WORK-
      ING WITH"; ANIMAL$
160   PRINT "ARE WE WORKING WITH <A>CRES
      OR <M>ILES?"
170   INPUT "TYPE <A> OR <M> AND PRESS
      <ENTER>";U$
180   IF U $="A" OR U$="a" THEN UNIT$="ACRES"
190   IF U $="M" OR U$="m" THEN UNIT$="MILES"
200   PRINT "WHAT IS THE TOTAL NUMBER OF
      SQUARE ";UNIT$;" WE ARE"
210   PRINT "WORKING WITH?";
215   INPUT N
220   PRINT "WHAT IS THE MINIMUM NUMBER OF
      SQUARE" UNIT$; "THAT EACH"; ANIMAL$
230   PRINT "REQUIRES FOR ITS SURVIVAL";
235   INPUT M
240   REM ****CALCULATE****
250   LET T=N/M
260   REM ****PRINT YOUR ANSWER***
270   PRINT "YOU MAY HAVE A MAXIMUM NUM-
      BER OF"
275   PRINT USING "##,###,###";T
280   PRINT" ANIMAL$;" ON THE ";N;";"UNIT$;"
      AVAILABLE."
290   REM ***GO AGAIN?***
300   PRINT:PRINT
310   PRINT "PRESS <A> IF YOU WANT TO TRY
      AGAIN, OR ANY OTHER KEY
      IF YOU DON'T";
320   INPUT AN$
330   IF AN$="A" OR AN$="a" THEN GOTO 130 ELSE
      END
```

[1] This program is designed for the TRS-80.

RUN

```
>RUN
THIS IS A SIMPLIFIED PROGRAM TO SHOW YOU
HOW COMPUTERS CAN GIVE PRACTICAL AN-
SWERS TO MATHEMATICAL MODELS IN EVERY-
DAY LIFE.

HERE WE ARE ASKED INFORMATION ABOUT HOW
MANY ANIMALS OF A GIVEN TYPE, CAN BE SUS-
TAINED BY A GIVEN PARCEL OF LAND. THIS IS DE-
TERMINED BY FIRST ENTERING THE MINIMUM
NUMBER OF UNITS EACH ANIMAL MUST HAVE TO
SURVIVE.
PLEASE PRESS ANY KEY TO CONTINUE
WHAT ANIMAL WILL YOU BE WORKING WITH?
HORSE
ARE WE WORKING WITH <A>CRES OR
<M>ILES?
TYPE <A> OR <M> AND PRESS <ENTER>? A
WHAT IS THE TOTAL NUMBER OF SQUARE ACRES
WE ARE WORKING WITH? 12
WHAT IS THE MINIMUM NUMBER OF ACRES THAT
EACH HORSE REQUIRES FOR
ITS SURVIVAL? 1
YOU MAY HAVE A MAXIMUM NUMBER OF
      12
HORSES ON THE 12 ACRES AVAILABLE.

PRESS <A> IF YOU WANT TO TRY AGAIN, OR ANY
OTHER KEY IF YOU DON'T? A
```

APPENDIX B

Water Conservation Program

PROGRAM

```
>LIST
10    REM**********
20    REM ***WATER CONSERVATION PRO-
      GRAM***
30    REM *** BY RICHARD WOLFERT *****
40    REM *** APRIL 1987 ***
50    REM**********
60    REM
70    REM
80    REM ***INPUT SECTION
90    CLS: PRINT "WATER CONSERVATION PRO-
      GRAM"
100   PRINT
110   PRINT "THIS IS A PROGRAM DESIGNED TO
      DEMONSTRATE HOW WATER"
120   PRINT "USE IN AN AREA CAN AFFECT THE
      NUMBER OF HOUSES,"
130   PRINT "FARMS OR FACTORIES THAT MAY BE
      BUILT IN THAT AREA AND"
140   PRINT "HOW COMPUTERS CAN HELP TO
      MAKE SUCH DECISIONS."
145   PRINT
150   PRINT "HERE WE MUST CONSIDER SUCH
      THINGS AS YEARLY"
160   PRINT "RAINFALL, ZONING REQUIRE-
      MENTS, AND NEEDS OF OTHER"
170   PRINT "INTERESTS SUCH AS FARMS AND
      FACTORIES, ETC., FOR 1"
180   PRINT "SQUARE MILE."
185   PRINT
190   PRINT "SOME GUIDELINES ARE GIVEN TO
      YOU, BUT YOU MAY CHANGE"
200   PRINT "THE NUMBERS IF YOU WISH AND
      SEE WHAT EFFECT THEY"
210   PRINT "HAVE IN THE OUTCOME."
215   PRINT
220   PRINT "THE NUMBERS USED HERE ARE NOT
      NECESSARILY REALISTIC"
230   PRINT "BUT ARE USED SIMPLY TO DEMON-
      STRATE THE IDEA OF THE"
240   PRINT "PROGRAM."
245   PRINT "PRESS ANY KEY TO CONTINUE"
250   GOSUB 2010
790   CLS:PRINT "WE PRESUME THAT THE
      GROUND WILL HOLD A GREAT"
800   PRINT "DEAL OF UNUSED WATER BETWEEN
      RAINS AND THAT THE"
810   PRINT "NUMBERS USED HERE ARE ONLY AV-
      ERAGES."
820   PRINT:PRINT "WE ALSO PRESUME THAT 50%
      OF THE WATER WILL"
830   PRINT "REMAIN TO ALLOW FOR EVAPORA-
      TION, LOCAL FORESTS AND PLANTS, RE-
      SERVES, ETC."
840   PRINT:PRINT" PRESS ANY KEY TO CON-
      TINUE"
850   GOSUB 2010
855   PRINT
857   PRINT
860   REM ***THIS SECTION GIVES PRESENT FIG-
      URES BUT ALLOWS
870   REM ***YOU TO CHANGE THEM.
880   REM
890   LET AR=34:ML=2.5:FC=3:FW
      =250000:FA=7:WF=18000
900   CLS:PRINT "HERE ARE SOME FIGURES THAT
      WILL WORK WITHOUT"
905   PRINT "CHANGES."
910   PRINT
920   PRINT" ANNUAL RAINFALL/SNOWFALL 34
      INCHES"
930   PRINT" MINIMUM LAND PER HOUSE 2.5 SQ.
      ACRES OR .004 SQ.MILES"
940   PRINT" (LOCAL ORDINANCE) THIS ALLOWS
      FOR 250 HOMES"
950   PRINT" PER SQUARE MILE OR "
960   PRINT" ABOUT 850 PEOPLE PER"
970   PRINT" SQUARE MILE"
975   PRINT" NUMBER OF FACTORIES-3"
980   PRINT" FACTORY WATER USE PER DAY-
      250000 GALLONS"
990   PRINT" NUMBER OF FARMS-7"
1000  PRINT" FARM WATER USE PER DAY-18,000
      GALLONS"
1010  PRINT "========================="
1015  PRINT
```

```
1020 PRINT" PRESS <0> FOR OK IF YOU WANT TO SEE THE COMPUTER WORK WITH THEM."
1030 PRINT"      OR"
1040 PRINT"PRESS <C> IF YOU WANT TO CHANGE THE FIGURES FIRST."
1050 PRINT"DON'T FORGET TO PRESS <ENTER> AFTER YOUR CHOICE.";
1060 INPUT AN$
1070     IF AN$="O" OR AN$="o" THEN GOTO 1280
1080     IF AN$="C" OR AN$="c" THEN GOTO 1090
1090 REM ***CHANGE ITEM AMOUNTS***
1100 CLS
1110 PRINT"THE SUGGESTED FIGURES FOR EACH ITEM ARE LISTED HERE"
1120 PRINT"1-ANNUAL RAIN/SNOWFALL -";AR;" INCHES"
1130 PRINT"2-MINIMUM LAND PER HOUSE -";ML;" SQUARE ACRES"
1140 PRINT"3-NUMBER OF FACTORIES -";FC
1150 PRINT"4-FACTORY WATER USE PER DAY -";FW;" GALLONS"
1160 PRINT"5-NUMBER OF FARMS -";FA
1170 PRINT"6-FARM WATER USE PER DAY -";WF;" GALLONS"
1175 PRINT"7-NO MORE CHANGES"
1180 PRINT"==========================="
1190 PRINT:INPUT"WHICH NUMBER DO YOU WANT TO CHANGE";NU
1200 ON NU GOTO 1210,1220,1230, 1240,1250,1260,1270
1210 INPUT"NEW ANNUAL RAIN/SNOWFALL (10-50 IN)";AR: GOTO 1100
1220 INPUT"NEW MIN. LAND PER HOUSE (1-10 ACRES)";ML: GOTO 1100
1230 INPUT"NEW NUMBER OF FACTORIES";FC" GOTO 1100
1240 INPUT"NEW FACTORY WATER USE/DAY (10,000-500,000 GALLONS";FW: GOTO 1100
1250 INPUT"NEW NUMBER OF FARMS";FA: GOTO 1100
1260 INPUT"NEW FARM WATER USE PER DAY";WF: GOTO 1100
1270 GOTO 1280
1280 REM***CALCULATION AND DISPLAY***
1290 LET DR=52000*AR: LET AU=DR/2
1300 LET WU=(FC*FW)+(FA*WF)+(640/ML)*3.4
1310 CLS
1320 PRINT:PRINT"THE AVERAGE DAILY WATER USE WITH"
1330 PRINT USING"THESE FIGURES IS: ##,###,###";WU; :PRINT"GALLONS"
1340 PRINT USING" ALLOWABLE USE: ##,###,###";AU;:PRINT"GALLONS PER DAY."
1350 IF WU > AU THEN GOTO 1360 ELSE 1410
1360 PRINT:PRINT "WATER USE TOO HIGH"
1370 PRINT "REWORK THE FIGURES AND TRY AGAIN."
1380 PRINT:PRINT"PRESS ANY KEY TO CONTINUE"
1390 GOSUB 2010
1400 GOTO 1090
1410 PRINT:PRINT "WATER USAGE IS WITHIN ALLOWABLE LIMITS, GOOD WORK!!"
1420 PRINT:PRINT"YOU MAY <T>RY AGAIN"
1430 PRINT"      OR"
1440 PRINT"    QUIT"
1450 INPUT AN$
1460 IF AN$="T" OR AN$="t" THEN GOTO 1090
1470 IF AN$="Q" OR AN$="q" THEN GOTO 1500
1480 GOTO 1450
1490 GOTO 1490
1500 PRINT:PRINT"PROGRAM ENDED"
2000 STOP
2010 REM INKEY FUNCTION
2020 LET A$=INKEY$:IFA$=" THEN 2010
2030 RETURN
READY

>RUN

          WATER CONSERVATION PROGRAM

THIS IS A PROGRAM DESIGNED TO DEMONSTRATE HOW WATER USE IN AN AREA CAN AFFECT THE NUMBER OF HOUSES, FARMS OR FACTORIES THAT MAY BE BUILT IN THAT AREA AND HOW COMPUTERS CAN HELP TO MAKE SUCH DECISIONS.

HERE WE MUST CONSIDER SUCH THINGS AS YEARLY RAINFALL, ZONING REQUIREMENTS, AND NEEDS OF OTHER INTERESTS SUCH AS FARMS AND FACTORIES, ETC., FOR 1 SQUARE MILE.

SOME GUIDELINES ARE GIVEN TO YOU, BUT YOU MAY CHANGE THE NUMBERS IF YOU WISH AND SEE WHAT EFFECT THEY HAVE IN THE OUTCOME.

THE NUMBERS USED HERE ARE NOT NECESSARILY REALISTIC BUT ARE USED SIMPLY TO DEMONSTRATE THE IDEA OF THE PROGRAM.

          PRESS ANY KEY TO CONTINUE

WE PRESUME THAT THE GROUND WILL HOLD A GREAT DEAL OF UNUSED WATER BETWEEN RAINS AND THAT THE NUMBERS USED HERE ARE ONLY AVERAGES.
```

WE ALSO PRESUME THAT 50% OF THE WATER WILL
REMAIN TO ALLOW FOR EVAPORATION, LOCAL FORESTS, AND

PRESS ANY KEY TO CONTINUE

HERE ARE SOME FIGURES THAT WILL WORK WITHOUT CHANGES.

ANNUAL RAINFALL	34 INCHES
MINIMUM LAND PER HOUSE	2.5 SQ ACRES OR 0.004 SQ. MILES
(LOCAL ORDINANCE)	THIS ALLOWS 250 HOMES PER SQUARE MILE OR ABOUT 850 PEOPLE PER SQUARE MILE
NUMBER OF FACTORIES	3
FACTORY WATER USE PER DAY	250000 GALLONS
NUMBER OF FARMS	7
FARM WATER USE PER DAY	18,000 GALLONS

==========

PRESS <O> FOR OK IF YOU WANT TO SEE THE RESULTS.
OR
PRESS <C> IF YOU WANT TO CHANGE THE FIGURES FIRST.
DON'T FORGET TO PRESS <ENTER> AFTER YOUR CHOICE.? O

THE AVERAGE DAILY WATER USE WITH

THESE FIGURES IS:	876,870 GALLONS.
ALLOWABLE USE:	884,000 GALLONS PER DAY.

WATER USAGE IS WITHIN ALLOWABLE LIMITS, GOOD WORK!!

YOU MAY <T>RY AGAIN
OR
<Q>UIT

?T
THE SUGGESTED FIGURES FOR EACH ITEM ARE LISTED HERE

1-ANNUAL RAIN/SNOWFALL	34 INCHES
2-MINIMUM LAND PER HOUSE	2.5 SQUARE ACRES
3-NUMBER OF FACTORIES	3
4-FACTORY WATER USE PER DAY	250000 GALLONS
5-NUMBER OF FARMS	7
6-FARM WATER USE PER DAY	18000 GALLONS
7-NO MORE CHANGES	

==========

WHICH NUMBER DO YOU WANT TO CHANGE? 2
NEW MIN. LAND PER HOUSE (1-10 ACRES)? 2
THE SUGGESTED FIGURES FOR EACH ITEM ARE LISTED HERE

1-ANNUAL RAIN/SNOWFALL	34 INCHES
2-MINIMUM LAND PER HOUSE	2 SQUARE MILES
3-NUMBER OF FACTORIES	3
4-FACTORY WATER USE PER DAY	250000 GALLONS
5-NUMBER OF FARMS	7
6-FARM WATER USE PER DAY	18000 GALLONS
7-NO MORE CHANGES	

==========

WHICH NUMBER DO YOU WANT TO CHANGE? 1
NEW ANNUAL RAIN/SNOWFALL (10-50 IN)? 14
THE SUGGESTED FIGURES FOR EACH ITEM ARE LISTED HERE

1-ANNUAL RAIN/SNOWFALL	14 INCHES
2-MINIMUM LAND PER HOUSE	2 SQUARE ACRES
3-NUMBER OF FACTORIES	3
4-FACTORY WATER USE PER DAY	250000 GALLONS
5-NUMBER OF FARMS	7
6-FARM WATER USE PER DAY	18000 GALLONS
7-NO MORE CHANGES	

==========

WHICH NUMBER DO YOU WANT TO CHANGE? 3
NEW NUMBER OF FACTORIES? 6
THE SUGGESTED FIGURES FOR EACH ITEM ARE LISTED HERE

1-ANNUAL RAIN/SNOWFALL	14 INCHES
2-MINIMUM LAND PER HOUSE	2 SQUARE ACRES
3-NUMBER OF FACTORIES	6
4-FACTORY WATER USE PER DAY	250000 GALLONS
5-NUMBER OF FARMS	7
6-FARM WATER USE PER DAY	18000 GALLONS
7-NO MORE CHANGES	

==========

WHICH NUMBER DO YOU WANT TO CHANGE? 4
NEW FACTORY WATER USE/DAY (10,000-500,000 GALLONS)? 20,000
?Extra ignored
THE SUGGESTED FIGURES FOR EACH ITEM ARE LISTED HERE

1-ANNUAL RAIN/SNOWFALL	14 INCHES
2-MINIMUM LAND PER HOUSE	2 SQUARE ACRES
3-NUMBER OF FACTORIES	6
4-FACTORY WATER USE PER DAY	20 GALLONS
5-NUMBER OF FARMS	7
6-FARM WATER USE PER DAY	18000 GALLONS
7-NO MORE CHANGES	

==========

WHICH NUMBER DO YOU WANT TO CHANGE? 5
NEW NUMBER OF FARMS? 4
THE SUGGESTED FIGURES FOR EACH ITEM ARE LISTED HERE

1-ANNUAL RAIN/SNOWFALL	14 INCHES
2-MINIMUM LAND PER HOUSE	2 SQUARE ACRES
3-NUMBER OF FACTORIES	6
4-FACTORY WATER USE PER DAY	20 GALLONS
5-NUMBER OF FARMS	4
6-FARM WATER USE PER DAY	18000 GALLONS
7-NO MORE CHANGES	

==========

WHICH NUMBER DO YOU WANT TO CHANGE? 6
NEW FARM WATER USE PER DAY? 16000
THE SUGGESTED FIGURES FOR EACH ITEM ARE LISTED HERE

1-ANNUAL RAIN/SNOWFALL	14 INCHES
2-MINIMUM LAND PER HOUSE	2 SQUARE ACRES
3-NUMBER OF FACTORIES	6
4-FACTORY WATER USE PER DAY	20 GALLONS
5-NUMBER OF FARMS	4
6-FARM WATER USE PER DAY	16000 GALLONS
7-NO MORE CHANGES	

==========

WHICH NUMBER DO YOU WANT TO CHANGE? 7

THE AVERAGE DAILY WATER USE WITH

THESE FIGURES IS:	65,208 GALLONS.
ALLOWABLE USE:	364,000 GALLONS PER DAY.

WATER USAGE IS WITHIN ALLOWABLE LIMITS, GOOD WORK!!

```
YOU MAY <T>RY AGAIN
OR
<Q>UIT

?Q

PROGRAM ENDED
Break in 2000
READY
```

APPENDIX C

Students with Handicapping Conditions

The Board of Regents, through revising the Part 100 Regulations of the Commissioner and the Action Plan, has made a strong commitment to integrating the education of students with handicapping conditions into the total school program. According to Section 100.2(s) "Each student with a handicapping condition, as such term is defined in Section 200.1(ii) of this Chapter, shall have access to the full range of programs and services set forth in this Part to the extent that such programs and services are appropriate to such student's special educational needs." Districts must have policies and procedures in place to make sure that students with handicapping conditions have equal opportunities to access diploma credits, courses, and requirements.

The majority of students with disabilities have the intellectual potential to master the curricula content requirements for a high school diploma. Most students who require special education attend regular education classes in conjunction with specialized instruction and/or related services. These students must attain the same academic standards as their nonhandicapped peers in order to meet these requirements. For this reason, it is very important that at all grade levels students with handicapping conditions receive instruction in the same content areas so as to receive the same informational base that will be required for proficiency on statewide testing programs and diploma requirements.

The teacher providing instruction through this syllabus/curriculum has the opportunity to provide an educational setting which will enable the students to explore their abilities and interests. Instruction could be provided to handicapped students either by teachers certified in this subject area or by special education teachers. Teachers certified in this subject area would be providing instruction to handicapped students who are recommended by the Committee on Special Education (CSE) as being able to benefit from instruction in a regular educational setting and are appropriately placed in this setting. Special education teachers may also provide this instruction to a class of students with handicapping conditions in a special class setting.

Teachers certified in the subject area should become aware of the needs of those students with handicapping conditions participating in their classes. Instructional techniques and materials must be modified to the extent appropriate to provide students with handicapping conditions the opportunity to meet diploma requirements. Information or assistance is available through special education teachers, administrators, the Committee on Special Education, or a student's IEP.

STRATEGIES FOR MODIFYING INSTRUCTIONAL TECHNIQUES AND MATERIALS

1. Prior to having a guest speaker or taking field trips, it may be helpful to structure the situation. Use of a checklist or a set of questions generated by the class will help students focus on relevant information. Accessibility for students with handicapping conditions should be considered when field trips are arranged.
2. The use of computer software may be appropriate for activities that require significant amounts of writing by students.
3. Students with handicapping conditions may use alternative testing techniques. The needed testing modifications must be identified in the student's Individualized Education Program. Both special and regular education teachers need to work in close cooperation so that the testing modifications can be used consistently throughout the student's program.
4. Identify, define, and pre-teach key vocabulary. Many terms in this syllabus are specific and may need continuous reinforcement for some students with handicapping conditions. It would also be helpful to provide a list of these key words to the special education teacher in order to provide additional reinforcement in the special educational setting.
5. Check periodically to determine student understanding of lectures, discussions, demonstrations, etc. and how this is related to the overall topic. Encourage students to express their understanding. It may be necessary to have small group discussions or work with partners to determine this.

6. Provide students and special education teachers with a tape of lectures that contain substantial new vocabulary content and of guest speakers for further review within their special education classes.
7. Assign a partner for the duration of a unit to a student as an additional resource to facilitate clarification of daily assignments, time lines for assignments, and access to daily class notes.
8. When assigning long-term projects/reports, provide a time line with benchmarks as indicators for completion of major project/report sections. Students who have difficulty with organizational skills and time sequence may need to see completion of sections to maintain the organization of a lengthy project/report.

Special education teachers providing this instruction must also become familiar with the goals and objectives of the curriculum. It is important that these teachers provide their students with the same or equivalent information contained in the curriculum.

Regardless of who provides the instruction, the cooperation between teachers of regular and special education programs is essential. It is important for the students as well as the total school environment.

ALTERNATIVE TESTING TECHNIQUES

Another consideration in assisting students with handicapping conditions to meet the requirements of regular education is the use of alternative testing techniques. Alternative testing techniques are modifications of testing procedures or formats which provide students with handicapping conditions equal opportunity to participate in testing situations. Such techniques provide the opportunity to demonstrate mastery of skills and attainment of knowledge without being limited or unfairly restricted by the existence of a handicapping condition.

The Committee on Special Education (CSE) is responsible for identifying and documenting a student's need for alternative testing techniques. This determination is made when a student is initially referred to CSE, is reviewed annually for as long as the student receives special education services, and is reviewed when the student is determined to no longer need special education services. **The modifications are to be used consistently throughout the student's educational program.** Principals ensure that students who have been identified by CSE as educationally handicapped are provided the alternative testing techniques which have been recommended by CSE and approved by the Board of Education.

Alternative testing techniques which have been specified on student IEPs for use by a student must be used consistently in both special and regular education settings. Regular classroom teachers should be aware of possible alternative testing techniques and should be skilled in their implementation.

The coordination and cooperation of the total school program will assist in providing the opportunity for a greater number of handicapped students to meet the requirements needed to pursue a high school diploma. The integrated provision of regular education programs, special education programs, remediation, alternative testing techniques, modified teacher techniques and materials, and access to credit through alternatives will assist in enabling such students to pursue high school diplomas to a greater degree. The teacher who provides instruction through this curriculum has a unique opportunity to assist such students in their individual goals.

Additional information on alternative testing modifications is available in the manual entitled *Alternative Techniques for Students with Handicapping Conditions*, which can be obtained from:

New York State Education Department
Office for Education of Children with
Handicapping Conditions
Room 1071 Education Building Annex
Albany, NY 12234

INFUSING AWARENESS OF PERSONS WITH DISABILITIES THROUGH CURRICULUM

In keeping with the concept of integration, the following subgoal of the Action Plan was established:

> *In all subject areas, revisions in the syllabi will include materials and activities related to* generic subgoals such as problem solving, reasoning skills, speaking, capacity to search for information, the use of libraries and increasing student awareness of and *information about the disabled.*

The purpose of this subgoal is to ensure that appropriate activities and materials are available to increase student awareness of disabilities and issues in regard to disabilities.

This curriculum, by design, includes information, activities, and materials regarding persons with handicapping conditions. Teachers are encouraged to include other examples as may be appropriate to their classrooms or the situation at hand. Teachers are also encouraged to assess the classroom environment to determine how the environment may contribute to student awareness of persons with disabilities.